Canonical description of the *D* = 10 superstring formulated in supertwistor space

Canonical description of the $D = 10$ superstring formulated in supertwistor space

D V Uvarov

National Science Center, Kharkov Institute of Physics and Technology, 61108 Kharkov, Ukraine

E-mail: duvarov@hotmail.com and uvarov@kipt.kharkov.ua

Abstract
Canonical description of the $D = 10$ superstring action involving supertwistor variables generalizing Penrose–Ferber supertwistors is developed. Primary and secondary constraints are identified and arranged into the first- and second-class sets. Dirac brackets are introduced and the deformation of the Poisson bracket algebra of the first-class constraints is studied. The role of the deformation parameter is played by α'.

PACS numbers: 11.25.−w, 11.30.Pb

1. Introduction

Twistors [1] and supertwistors [2] are known to find one of the interesting applications in describing the models of point-like and extended relativistic objects [3–25]. Such (super)twistor formulations based on the introduction of commuting spinor variables represent a valuable alternative to the conventional (super)space formulation and allow us to overcome the problem of handling κ-symmetry and streamline the covariant quantization in the case of (super)particle models. Thus incorporating supertwistors into the string theory could also be useful in the quest for a solution of the long-standing problem of Green–Schwarz (GS) superstring covariant quantization. However, twistor description of (supersymmetric) models of extended objects attracted much less attention until the twistor strings [26, 27][1] were proposed in the context of gauge fields/string correspondence. Although twistor string models appear to be interesting objects for study and have stimulated recent progress in perturbative (super-)Yang–Mills and gravitation [40], they differ from the GS superstrings.

In recent years the progress in solving the problem of superstring covariant quantization was mainly due to Berkovits formalism [41]. Its key ingredient is the BRST operator involving the ten-dimensional pure spinor field with commuting complex components that plays the

---

[1] See also [28–30] and [31–39] for other twistor string models corresponding to 4D gauge theories and supergravities.

role of the ghost field for the fermionic constraints of the GS superstring and can also be viewed as half of the twistor [42]. The main advantage of the Berkovits approach is the possibility of finding covariant expressions for the string scattering amplitudes [41, 43, 44]. It is worthwhile to note that originally pure spinors appeared as the worldsheet superpartners of superspace Grassmann coordinates θ^α in the heterotic string formulation [45, 46] with $n = 2$ local worldsheet supersymmetry[2]. In the formulation of [45, 46], pure spinors also admit interpretation as the pair of elements from the basis in the auxiliary spinor space extending $D = 10, N = 1$ superspace. The interesting problem of finding the relation between the Berkovits model and the GS superstrings or their classically equivalent reformulations has been being investigated since the year 2000 (see [49] and references therein).

Taking into account the above-mentioned results that indicate an important role played by spinors/twistors in the quantum theory of superparticles and superstrings we have started to study the twistor formulation for GS superstrings in dimensions $D = 4, 6, 10$ [50, 51] aiming at getting novel insights into the covariant qunatization problem. Since commuting spinor variables—the necessary ingredients of twistors—are absent in the original GS superstring action we considered, similarly to the twistor transform procedure for (super)particles, the classically equivalent first-order superstring action [52], where such bosonic spinors form the basis in auxiliary space—the space of Lorentz harmonics [53–58]. Note that $D = 4$ spinor Lorentz harmonics are nothing but the normalized Newman–Penrose dyad [59]. The presence of Lorentz harmonic variables in the Lagrangian allows us to realize κ-symmetry transformations in the irreducible form. In the formulations of [45, 46] pure spinors provide, in a similar way, irreducible realization of the part of κ-symmetries. Detailed discussion of the relation between the superstring formulations involving Lorentz harmonics and those with local worldsheet supersymmetry can be found in [60].

The supertwistors appearing upon the twistor transform of the first-order action [52] coincide for the $D = 4$ case with those introduced by Ferber [2][3], while in higher dimensions [20, 29] they realize the fundamental representation of the (generalized) superconformal group, include spinor harmonics as their projectional parts, and the Grassmann-odd components of supertwistors are represented by the Lorentz scalars, which is an attractive feature from the perspective of fixing the gauge freedom related to the κ-symmetry. In [50, 51] we found the supertwistor representation for $D = 4, 6, 10$ superstring Lagrangian characterized by the nondegenerate kinetic term for the supertwistor components, derived the equations of motion, and obtained the supertwistor realization of the κ-symmetry transformations.

Since the superstring Lagrangian after the twistor transform is nonlinear, like in the spacetime formulation, and the supertwistors are constrained variables, the canonical formalism appears to be the most suitable one for further investigation. That is why here we pass to the canonical description of the $D = 10$ superstring model formulated in terms of supertwistors. In section 2, we identify the constraints that arise in the process of transition to the Hamiltonian formulation and classify them on the first- and second-class ones. Up to that step our consideration can be viewed as the twistor counterpart of the canonical treatment of Lorentz-harmonic superstring in the superspace formulation [52]. Then in section 3 we proceed to propose the basis for the second-class constraints for which the Dirac matrix

---

[2] Pure spinors also turned out to be useful in the study of the geometry of superfield constraints in super-Yang–Mills and supergravity theories [47, 48].
[3] Recently, an alternative to Ferber construction of the supertwistors has been proposed [61], where only conformal super-Poincare symmetry is manifest and odd supertwistor components are given by the complex Lorentz vectors related to those appearing in the particle and string models with worldline/worldsheet supersymmetry. Since superstring Lagrangian, due to the presence of the dimensionful tension parameter, is not invariant under conformal transformations, it could be of interest to consider its formulation in terms of such alternative supertwistors or their higher dimensional generalizations [58, 62].

acquires a block diagonal structure on the constraint shell, introduce the Dirac brackets (DB), and evaluate DB algebra of the first-class constraints.

2. Total Hamiltonian and the first-class constraints

Based on the classification of superconformal algebras in various dimensions [63] it was suggested in [20, 51] to define a $D = 10$ supertwistor as realizing the fundamental representation of the $OSp(32|1)$ supergroup

$$Z^{\Lambda} = (\mu^{\alpha}, v_{\alpha}, \eta). \tag{1}$$

Thus, it is composed of the primary spinor μ^{α} and projectional v_{α} parts that are 16-component MW spinors of opposite chiralities and the Grassmann-odd scalar η. Such definition generalizes the basic property of Ferber supertwistors [2] to realize the fundamental representation of $SU(2, 2|N)$ locally isomorphic to the N-extended superconformal group in four dimensions. Twistor transform for the $D = 10$ superstring in the formulation with irreducible realization of the κ-symmetry leads one to consider two sets of the supertwistors

$$Z_A^{\Lambda+} = \left(\mu_A^{\alpha+}, v_{\alpha A}^+, \eta_A^+\right), \qquad Z_{\dot A}^{\Lambda-} = \left(\mu_{\dot A}^{\alpha-}, v_{\alpha \dot A}^-, \eta_{\dot A}^-\right) \tag{2}$$

whose projectional parts are identified with the spinor harmonic matrix $v_{\alpha}^{(\alpha)} = \left(v_{\alpha A}^+, v_{\alpha \dot A}^-\right) \in$ Spin(1, 9) decomposed into two blocks carrying $SO(1, 1)$ indices $\pm$ and transforming in the spinor representations of Spin(8) $A, \dot A = 1, \ldots, 8$ in accordance with that the embedding of the string worldsheet into the $D = 10$ spacetime spontaneously breaks $SO(1, 9)$ symmetry down to $SO(1, 1) \times SO(8)$. The $D = 10$ generalization of the Penrose–Ferber incidence relations[4]

$$\mu_A^{\alpha+} = (X^{\alpha\beta} - 8i\theta^{\alpha}\theta^{\beta})v_{\beta A}^+, \qquad \eta_A^+ = 4v_{\alpha A}^+ \theta^{\alpha}$$
$$\mu_{\dot A}^{\alpha-} = (X^{\alpha\beta} - 8i\theta^{\alpha}\theta^{\beta})v_{\beta \dot A}^-, \qquad \eta_{\dot A}^- = 4v_{\alpha \dot A}^- \theta^{\alpha} \tag{3}$$

involves an arbitrary 16×16 matrix $X^{\alpha\beta} = x^m \tilde\sigma_m^{\alpha\beta} + z^{m_1 m_2 m_3} \tilde\sigma_{m_1 m_2 m_3}^{\alpha\beta} + z^{m_1 \ldots m_5} \tilde\sigma_{m_1 \ldots m_5}^{\alpha\beta}$ that contains, except for $D = 10$ Minkowski coordinates x^m, antisymmetric tensor coordinates $z^{m_1 m_2 m_3}$ and $z^{m_1 \ldots m_5}$ associated with tensor generators of $OSp(32|1)$. Since our goal is to describe the superstring in $D = 10$ Minkowski superspace, the dependence on such tensor coordinates has to be removed by the constraints

$$N_{AB}^{+2} = Z_A^{\Lambda+} G_{\Lambda\Sigma} Z_B^{\Sigma+} \approx 0, \quad N_{\dot A \dot B}^{-2} = Z_{\dot A}^{\Lambda-} G_{\Lambda\Sigma} Z_{\dot B}^{\Sigma-} \approx 0, \quad N_{A\dot A} = Z_A^{\Lambda+} G_{\Lambda\Sigma} Z_{\dot A}^{\Sigma-} \approx 0, \tag{4}$$

where

$$G_{\Lambda\Sigma} = \begin{pmatrix} 0 & \delta_{\alpha}^{\beta} & 0 \\ -\delta_{\beta}^{\alpha} & 0 & 0 \\ 0 & 0 & -i \end{pmatrix} \tag{5}$$

is the $OSp(32|1)$ invariant orthosymplectic metric, and

$$N_{m_1 \ldots m_5} = \sigma_{m_1 \ldots m_5 \alpha\beta}\left(\mu_A^{\alpha+} v_A^{\beta-} + \mu_{\dot A}^{\alpha-} v_{\dot A}^{\beta+}\right) \approx 0. \tag{6}$$

The latter constraints involve the inverse spinor harmonic matrix

$$v_{(\alpha)}^{\alpha} = \left(v_A^{\alpha-}, v_{\dot A}^{\alpha+}\right): \qquad v_{(\alpha)}^{\alpha} v_{\alpha}^{(\beta)} = \delta_{(\alpha)}^{(\beta)}. \tag{7}$$

The first-order action of the $D = 10, N = 1$ superstring reformulated in terms of supertwistors (2) was found in [51]. For the transition to the canonical formulation, it is

[4] Such $D = 10$ incidence relations with $X^{\alpha\beta}$ matrix involving solely spacetime coordinates x^m contribution have been proposed in [58].

convenient instead of the zweibein $e^{\pm 2}_\mu(\xi)$ and its inverse $e^{\mu \pm 2}(\xi)$, upon which the action depends nonlinearly, to introduce worldsheet vector densities $\rho^{\pm 2} = (\alpha')^{1/2} e e^{\mu \pm 2}, e = \det(e^{\pm 2}_\mu)$ [52], so that the superstring action in the twistor formulation acquires the form

$$S = -\frac{1}{2\alpha'} \int d^2\xi \left(\rho^{\mu+2}\omega^{-2}_\mu + \rho^{\mu-2}\omega^{+2}_\mu + \varepsilon_{\mu\nu}\rho^{\mu-2}\rho^{\nu+2} \right) + S_{\text{WZ}}, \tag{8}$$

where S_{WZ} is the WZ term in the twistor representation given by

$$S_{\text{WZ}} = \frac{is}{\alpha'} \int d^2\xi \left(\frac{1}{2}\omega^{+2}(d) \wedge \varphi^{-2}(d) + \frac{1}{2}\omega^{-2}(d) \wedge \varphi^{+2}(d) - \omega^I(d) \wedge \varphi^I(d) \right). \tag{9}$$

In expressions (8) and (9) $\xi^\mu = (\tau, \sigma)$ are the worldsheet local coordinates, α' is the Regge slope parameter, and $s = \pm 1$ indicates the arbitrariness in the definition of the WZ part of the action. The action depends on the worldsheet projections of the 1-forms

$$\omega^{+2}(d) = \tfrac{1}{8} dZ^{\Lambda+}_A G_{\Lambda\Sigma} Z^{\Sigma+}_A, \qquad \omega^{-2}(d) = \tfrac{1}{8} dZ^{\Lambda-}_{\dot A} G_{\Lambda\Sigma} Z^{\Sigma-}_{\dot A},$$
$$\omega^I(d) = \tfrac{1}{16} \gamma^I_{A\dot A}\left(dZ^{\Lambda+}_A G_{\Lambda\Sigma} Z^{\Sigma-}_{\dot A} + dZ^{\Lambda-}_{\dot A} G_{\Lambda\Sigma} Z^{\Sigma+}_A \right), \tag{10}$$

and also

$$\varphi^{+2}(d) = \tfrac{1}{8}\mathcal{D}\eta^+_A \eta^+_A, \qquad \varphi^{-2}(d) = \tfrac{1}{8}\mathcal{D}\eta^-_{\dot A} \eta^-_{\dot A}, \qquad \varphi^I(d) = \tfrac{1}{16}\gamma^I_{A\dot A}\left(\mathcal{D}\eta^+_A \eta^-_{\dot A} + \mathcal{D}\eta^-_{\dot A} \eta^+_A \right). \tag{11}$$

$SO(1,9)$ covariant differentials of the odd supertwistor components are defined as

$$\mathcal{D}\eta^+_A = d\eta^+_A + \tfrac{1}{4}\Omega^{+2-2}(d)\eta^+_A - \tfrac{1}{2}\Omega^{+2I}(d)\gamma^I_{A\dot A}\eta^-_{\dot A} - \tfrac{1}{4}\Omega^{IJ}(d)\gamma^{IJ}_{AB}\eta^+_B,$$
$$\mathcal{D}\eta^-_{\dot A} = d\eta^-_{\dot A} - \tfrac{1}{4}\Omega^{+2-2}(d)\eta^-_{\dot A} - \tfrac{1}{2}\Omega^{-2I}(d)\tilde\gamma^I_{\dot A A}\eta^+_A - \tfrac{1}{4}\Omega^{IJ}(d)\tilde\gamma^{IJ}_{\dot A \dot B}\eta^-_{\dot B}, \tag{12}$$

where $\gamma^I_{A\dot A}$ are 8D chiral γ-matrices, satisfying the condition that $\gamma^I_{A\dot B}\tilde\gamma^J_{\dot B B} + (I \leftrightarrow J) = 2\delta^{IJ}\delta_{AB}, \gamma^{IJ}_{AB} = \tfrac{1}{2}\left(\gamma^I_{A\dot A}\tilde\gamma^J_{\dot A B} - \gamma^J_{A\dot A}\tilde\gamma^I_{\dot A B}\right)$ and $\tilde\gamma^{IJ}_{\dot A \dot B} = \tfrac{1}{2}\left(\tilde\gamma^I_{\dot A A}\gamma^J_{A\dot B} - \tilde\gamma^J_{\dot A A}\gamma^I_{A\dot B}\right)$ are the Spin(8) generators in the c and s representations, and contain $SO(1,1) \times SO(8)$ split components of the trivial $SO(1,9)$ connection constructed out of the spinor harmonics $v^{(\alpha)}_\alpha = \left(v^+_{\alpha A}, v^-_{\alpha \dot A}\right)$ and their inverse $v^\alpha_{(\alpha)} = \left(v^{\alpha-}_A, v^{\alpha+}_{\dot A}\right)$,

$$\Omega^{+2-2}(d) = \tfrac{1}{4}\left(dv^-_{\alpha\dot A}v^{\alpha+}_{\dot A} - dv^+_{\alpha A}v^{\alpha-}_A \right),$$
$$\Omega^{+2I}(d) = \tfrac{1}{4} dv^+_{\alpha A}\gamma^I_{A\dot A}v^{\alpha+}_{\dot A}, \qquad \Omega^{-2I}(d) = \tfrac{1}{4} dv^-_{\alpha\dot A}\tilde\gamma^I_{\dot A A}v^{\alpha-}_A, \tag{13}$$
$$\Omega^{IJ}(d) = \tfrac{1}{8}\left(dv^+_{\alpha A}\gamma^{IJ}_{AB}v^{\alpha-}_B + dv^-_{\alpha\dot A}\tilde\gamma^{IJ}_{\dot A \dot B}v^{\alpha+}_{\dot B} \right).$$

Passing to the canonical formulation we introduce the momenta densities

$$P_{\mathfrak{M}}(\tau,\sigma) = \frac{\vec\delta S}{\delta \dot Q^{\mathfrak{M}}(\tau,\sigma)} = \left\{ p^-_{(\mu)\alpha A}, p^+_{(\mu)\alpha\dot A}, p^{\alpha-}_{(v)A}, p^{\alpha+}_{(v)\dot A}, \pi^-_A, \pi^+_{\dot A}, p^+_{(v)\alpha A}, p^-_{(v)\alpha\dot A}, P^{\pm 2}_\mu \right\} \tag{14}$$

conjugate to the string coordinates

$$Q^{\mathfrak{M}}(\tau,\sigma) = \left\{ \mu^{\alpha+}_A, \mu^{\alpha-}_{\dot A}, v^+_{\alpha A}, v^-_{\alpha\dot A}, \eta^+_A, \eta^-_{\dot A}, v^{\alpha-}_A, v^{\alpha+}_{\dot A}, \rho^{\mu\pm 2} \right\} \tag{15}$$

on the Poisson brackets (PB)

$$\{P_{\mathfrak{M}}(\sigma), Q^{\mathfrak{N}}(\sigma')\} = \delta^{\mathfrak{N}}_{\mathfrak{M}}\delta(\sigma - \sigma'). \tag{16}$$

From the definition of momenta densities conjugate to primary spinor parts of supertwistors, there follow the constraints,

$$\Phi^-_{\alpha A}(\sigma) = p^-_{(\mu)\alpha A} + \frac{1}{16\alpha'}\left(\rho^{\tau-2} - is\varphi^{-2}_\sigma \right)v^+_{\alpha A} + \frac{is}{16\alpha'}\varphi^I_\sigma \gamma^I_{A\dot A}v^-_{\alpha\dot A} \approx 0,$$
$$\Phi^+_{\alpha\dot A}(\sigma) = p^+_{(\mu)\alpha\dot A} + \frac{1}{16\alpha'}\left(\rho^{\tau+2} - is\varphi^{+2}_\sigma \right)v^-_{\alpha\dot A} + \frac{is}{16\alpha'}\varphi^I_\sigma \tilde\gamma^I_{\dot A A}v^+_{\alpha A} \approx 0 \tag{17}$$

4

and analogously from the definition of momenta densities conjugate to anticommuting supertwistor components, there stem the fermionic constraints that can be presented as

$$
\begin{aligned}
D_A^-(\sigma) = {} & \pi_A^- + \frac{1}{16\alpha'}\left(\mathrm{i}s\omega_\sigma^{-2} - s\varphi_\sigma^{-2} - \mathrm{i}\rho^{\tau-2}\right)\eta_A^+ + \frac{s}{16\alpha'}\left(\varphi_\sigma^I - \mathrm{i}\omega_\sigma^I\right)\gamma_{A\dot A}^I\eta_{\dot A}^- \\
& + \frac{\mathrm{i}}{2}\eta_B^+\left(v_B^{\alpha-}p_{(\mu)\alpha A}^- - v_A^{\alpha-}p_{(\mu)\alpha B}^-\right) + \frac{\mathrm{i}}{2}\eta_{\dot B}^-\left(v_{\dot B}^{\alpha+}p_{(\mu)\alpha A}^- - v_A^{\alpha-}p_{(\mu)\alpha\dot B}^+\right) \approx 0,
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
D_{\dot A}^+(\sigma) = {} & \pi_{\dot A}^+ + \frac{1}{16\alpha'}\left(\mathrm{i}s\omega_\sigma^{+2} - s\varphi_\sigma^{+2} - \mathrm{i}\rho^{\tau+2}\right)\eta_{\dot A}^- + \frac{s}{16\alpha'}\left(\varphi_\sigma^I - \mathrm{i}\omega_\sigma^I\right)\tilde\gamma_{\dot A A}^I\eta_A^+ \\
& + \frac{\mathrm{i}}{2}\eta_B^+\left(v_B^{\alpha-}p_{(\mu)\alpha\dot A}^+ - v_{\dot A}^{\alpha+}p_{(\mu)\alpha B}^-\right) + \frac{\mathrm{i}}{2}\eta_{\dot B}^-\left(v_{\dot B}^{\alpha+}p_{(\mu)\alpha\dot A}^+ - v_{\dot A}^{\alpha+}p_{(\mu)\alpha\dot B}^+\right) \approx 0.
\end{aligned}
$$

In the harmonic sector, one finds the primary constraints arising from the definition of momenta conjugate to spinor harmonics $v_{\alpha A}^+$ and $v_{\alpha\dot A}^-$,

$$
\begin{aligned}
T_A^{\alpha-}(\sigma) = {} & p_{(v)A}^{\alpha-} + \frac{1}{16\alpha'}\left(\mathrm{i}s\varphi_\sigma^{-2} - \rho^{\tau-2}\right)\mu_A^{\alpha+} - \frac{\mathrm{i}s}{16\alpha'}\varphi_\sigma^I\gamma_{A\dot A}^I\mu_{\dot A}^{\alpha-} + \frac{\mathrm{i}s}{128\alpha'}\omega_\sigma^I(\eta^+\gamma^I\eta^-)v_A^{\alpha-} \\
& + \frac{\mathrm{i}s}{256\alpha'}\left[\frac{1}{2}\omega_\sigma^{-2}(\eta^+\gamma^{IJ}\eta^+) + \frac{1}{2}\omega_\sigma^{+2}(\eta^-\tilde\gamma^{IJ}\eta^-) - \omega_\sigma^K(\eta^+\gamma^{IJK}\eta^-)\right]\gamma_{AB}^{IJ}v_B^{\alpha-} \\
& + \frac{\mathrm{i}s}{128\alpha'}\left[\omega_\sigma^{-2}(\eta^+\gamma^I\eta^-) + \omega_\sigma^J(\eta^-\tilde\gamma^{IJ}\eta^-)\right]\gamma_{A\dot A}^I v_{\dot A}^{\alpha+} \approx 0,
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
T_{\dot A}^{\alpha+}(\sigma) = {} & p_{(v)\dot A}^{\alpha+} + \frac{1}{16\alpha'}\left(\mathrm{i}s\varphi_\sigma^{+2} - \rho^{\tau+2}\right)\mu_{\dot A}^{\alpha-} - \frac{\mathrm{i}s}{16\alpha'}\varphi_\sigma^I\tilde\gamma_{\dot A A}^I\mu_A^{\alpha+} - \frac{\mathrm{i}s}{128\alpha'}\omega_\sigma^I(\eta^+\gamma^I\eta^-)v_{\dot A}^{\alpha+} \\
& + \frac{\mathrm{i}s}{256\alpha'}\left[\frac{1}{2}\omega_\sigma^{-2}(\eta^+\gamma^{IJ}\eta^+) + \frac{1}{2}\omega_\sigma^{+2}(\eta^-\tilde\gamma^{IJ}\eta^-) - \omega_\sigma^K(\eta^+\gamma^{IJK}\eta^-)\right]\tilde\gamma_{\dot A\dot B}^{IJ}v_{\dot B}^{\alpha+} \\
& - \frac{\mathrm{i}s}{128\alpha'}\left[\omega_\sigma^{+2}(\eta^+\gamma^I\eta^-) - \omega_\sigma^J(\eta^+\gamma^{IJ}\eta^+)\right]\tilde\gamma_{\dot A A}^I v_A^{\alpha-} \approx 0,
\end{aligned}
\tag{20}
$$

and their inverse $v_A^{\alpha-}$, $v_{\dot A}^{\alpha+}$,

$$
p_{(v)\alpha A}^+(\sigma) \approx 0, \qquad p_{(v)\alpha\dot A}^-(\sigma) \approx 0.
\tag{21}
$$

The momenta densities for $\rho^{\mu\pm2}$ also enter the set of primary constraints

$$
P_\tau^{\pm2}(\sigma) \approx 0,
\tag{22}
$$

$$
P_\sigma^{\pm2}(\sigma) \approx 0.
\tag{23}
$$

Besides that 16×16 spinor harmonic matrix $v_\alpha^{(\alpha)}$ is constrained by 211 relations [52, 58],

$$
\begin{aligned}
& n_m^{(k)}v_\alpha^{(\alpha)}\tilde\sigma^{mm_1...m_4\alpha\beta}v_\beta^{(\beta)}\sigma_{(k)(\alpha)(\beta)} \approx 0, \\
& n_m^{+2}n^{m-2} - 2 = \frac{1}{64}\left(v_{\alpha A}^+\tilde\sigma_m^{\alpha\beta}v_{\beta A}^+\right)\left(v_{\gamma\dot A}^-\tilde\sigma^{m\gamma\delta}v_{\delta\dot A}^-\right) - 2 \approx 0,
\end{aligned}
\tag{24}
$$

reducing its contents to 45 independent components equal to the dimension of the Spin(1,9) group. Defining relations for the inverse harmonics, when considered as independent degrees of freedom,

$$
v_\alpha^{(\alpha)}v_{(\beta)}^\alpha - \delta_{(\beta)}^{(\alpha)} \approx 0
\tag{25}
$$

should also be treated as constraints, as well as the twistor constraints (4) and (6).

It was shown in [52] that the canonical analysis simplifies essentially if one excludes from the set of constraints harmonicity conditions (24) and (25) and appropriate projections of the

harmonic momenta (19)–(21), forming on PB conjugate pairs of the second-class constraints, by introducing corresponding DB. A suggestive feature of the DB is that they coincide with the PB for the subspace of the phase space defined by the primary constraints (17), (18), (22) and (23) and the projections of harmonic momenta

$$
\begin{aligned}
M^{+2-2}(\sigma) = {}& v_{\alpha A}^{+} p_{(v)A}^{\alpha -} - v_{\alpha \dot A}^{-} p_{(v)\dot A}^{\alpha +} - v_{A}^{\alpha -} p_{(v)\alpha A}^{+} + v_{\dot A}^{\alpha +} p_{(v)\alpha \dot A}^{-} + \mu_{A}^{\alpha +} p_{(\mu)\alpha A}^{-} \\
& - \mu_{\dot A}^{\alpha -} p_{(\mu)\alpha \dot A}^{+} + \eta_{A}^{+} \pi_{A}^{-} - \eta_{\dot A}^{-} \pi_{\dot A}^{+} \approx 0,
\end{aligned} \tag{26}
$$

$$
M^{+2I}(\sigma) = -v_{\alpha A}^{+} \gamma_{A\dot A}^{I} p_{(v)\dot A}^{\alpha +} + v_{\dot A}^{\alpha +} \tilde\gamma_{\dot A A}^{I} p_{(v)\alpha A}^{+} - \mu_{A}^{\alpha +} \gamma_{A\dot A}^{I} p_{(\mu)\alpha \dot A}^{+} - \eta_{A}^{+} \gamma_{A\dot A}^{I} \pi_{\dot A}^{+} \approx 0, \tag{27}
$$

$$
M^{-2I}(\sigma) = -v_{\alpha \dot A}^{-} \tilde\gamma_{\dot A A}^{I} p_{(v)A}^{\alpha -} + v_{A}^{\alpha -} \gamma_{A\dot A}^{I} p_{(v)\alpha \dot A}^{-} - \mu_{\dot A}^{\alpha -} \tilde\gamma_{\dot A A}^{I} p_{(\mu)\alpha A}^{-} - \eta_{\dot A}^{-} \tilde\gamma_{\dot A A}^{I} \pi_{A}^{-} \approx 0, \tag{28}
$$

$$
\begin{aligned}
M^{IJ}(\sigma) = -\tfrac{1}{2}\big(& v_{\alpha A}^{+} \gamma_{AB}^{IJ} p_{(v)B}^{\alpha -} + v_{\alpha \dot A}^{-} \tilde\gamma_{\dot A \dot B}^{IJ} p_{(v)\dot B}^{\alpha +} + v_{A}^{\alpha -} \gamma_{AB}^{IJ} p_{(v)\alpha B}^{+} + v_{\dot A}^{\alpha +} \tilde\gamma_{\dot A \dot B}^{IJ} p_{(v)\alpha \dot B}^{-} \\
& + \mu_{A}^{\alpha +} \gamma_{AB}^{IJ} p_{(\mu)\alpha B}^{-} + \mu_{\dot A}^{\alpha -} \tilde\gamma_{\dot A \dot B}^{IJ} p_{(\mu)\alpha \dot B}^{+} + \eta_{A}^{+} \gamma_{AB}^{IJ} \pi_{B}^{-} + \eta_{\dot A}^{-} \tilde\gamma_{\dot A \dot B}^{I} \pi_{\dot B}^{+}\big) \approx 0,
\end{aligned} \tag{29}
$$

complementing those that define the DB. Constraints (26)–(29) also include the contributions of other supertwistor components and their conjugate momenta and coincide with the linear combinations of primary constraints (17)–(21),

$$
\begin{aligned}
\hat M^{+2-2}(\sigma) = {}& v_{\alpha A}^{+} T_{A}^{\alpha -} - v_{\alpha \dot A}^{-} T_{\dot A}^{\alpha +} - v_{A}^{\alpha -} p_{(v)\alpha A}^{+} + v_{\dot A}^{\alpha +} p_{(v)\alpha \dot A}^{-} + \mu_{A}^{\alpha +} \Phi_{\alpha A}^{-} \\
& - \mu_{\dot A}^{\alpha -} \Phi_{\alpha \dot A}^{+} + \eta_{A}^{+} D_{A}^{-} - \eta_{\dot A}^{-} D_{\dot A}^{+} \approx 0, \\
\hat M^{+2I}(\sigma) = {}& -v_{\alpha A}^{+} \gamma_{A\dot A}^{I} T_{\dot A}^{\alpha +} + v_{\dot A}^{\alpha +} \tilde\gamma_{\dot A A}^{I} p_{(v)\alpha A}^{+} - \mu_{A}^{\alpha +} \gamma_{A\dot A}^{I} \Phi_{\alpha \dot A}^{+} - \eta_{A}^{+} \gamma_{A\dot A}^{I} D_{\dot A}^{+} \approx 0, \\
\hat M^{-2I}(\sigma) = {}& -v_{\alpha \dot A}^{-} \tilde\gamma_{\dot A A}^{I} T_{A}^{\alpha -} + v_{A}^{\alpha -} \gamma_{A\dot A}^{I} p_{(v)\alpha \dot A}^{-} - \mu_{\dot A}^{\alpha -} \tilde\gamma_{\dot A A}^{I} \Phi_{\alpha A}^{-} - \eta_{\dot A}^{-} \tilde\gamma_{\dot A A}^{I} D_{A}^{-} \approx 0, \\
\hat M^{IJ}(\sigma) = {}& -\tfrac{1}{2}\big(v_{\alpha A}^{+} \gamma_{AB}^{IJ} T_{B}^{\alpha -} + v_{\alpha \dot A}^{-} \tilde\gamma_{\dot A \dot B}^{IJ} T_{\dot B}^{\alpha +} + v_{A}^{\alpha -} \gamma_{AB}^{IJ} p_{(v)\alpha B}^{+} + v_{\dot A}^{\alpha +} \tilde\gamma_{\dot A \dot B}^{IJ} p_{(v)\alpha \dot B}^{-} \\
& + \mu_{A}^{\alpha +} \gamma_{AB}^{IJ} \Phi_{\alpha B}^{-} + \mu_{\dot A}^{\alpha -} \tilde\gamma_{\dot A \dot B}^{IJ} \Phi_{\alpha \dot B}^{+} + \eta_{A}^{+} \gamma_{AB}^{IJ} D_{B}^{-} + \eta_{\dot A}^{-} \tilde\gamma_{\dot A \dot B}^{I} D_{\dot B}^{+}\big) \approx 0
\end{aligned} \tag{30}
$$

modulo the twistor constraints (4) and (6). These so-called covariant momentum densities (26)–(29) satisfy on PB the relations of $SO(1,9)$ algebra and are the generators of infinitesimal local $SO(1,9)$ transformations acting on the supertwistor variables.

Since the twistor formulation is characterized by the presence of twistor constraints (4) and (6) it is helpful to introduce DB that take them into account as well. Considering the projections of constraints (17),

$$
\begin{aligned}
& \Phi_{\dot A \dot B}^{+2}(\sigma) = v_{\dot A}^{\alpha +} \Phi_{\alpha \dot B}^{+} - (\dot A \leftrightarrow \dot B) \approx 0, \qquad \Phi_{AB}^{-2}(\sigma) = v_{A}^{\alpha -} \Phi_{\alpha B}^{-} - (A \leftrightarrow B) \approx 0, \\
& \Phi_{A\dot B}(\sigma) = v_{A}^{\alpha -} \Phi_{\alpha \dot B}^{+} - v_{\dot B}^{\alpha +} \Phi_{\alpha A}^{-} \approx 0, \\
& \Phi^{m_1 \ldots m_5}(\sigma) = v_{\alpha A}^{+} \tilde\sigma^{m_1 \ldots m_5 \alpha \beta} \Phi_{\beta A}^{-} + v_{\alpha \dot A}^{-} \tilde\sigma^{m_1 \ldots m_5 \alpha \beta} \Phi_{\alpha \dot A}^{+} \approx 0
\end{aligned} \tag{31}
$$

conjugate to the twistor constraints (4) and (6) on the PB,

$$
\{\Phi_{\dot A \dot B}^{+2}(\sigma), N_{\dot C \dot D}^{-2}(\sigma')\} = 2(\delta_{\dot A \dot D} \delta_{\dot B \dot C} - \delta_{\dot A \dot C} \delta_{\dot B \dot D})\delta(\sigma - \sigma'), \tag{32}
$$

$$
\{\Phi_{AB}^{-2}(\sigma), N_{CD}^{+2}(\sigma')\} = 2(\delta_{AD} \delta_{BC} - \delta_{AC} \delta_{BD})\delta(\sigma - \sigma'), \tag{33}
$$

$$
\{\Phi_{A\dot B}(\sigma), N_{C\dot D}(\sigma')\} = -2\delta_{AC} \delta_{\dot B \dot D}\delta(\sigma - \sigma'), \tag{34}
$$

$$
\{\Phi^{m_1 \ldots m_5}(\sigma), N^{n_1 \ldots n_5}(\sigma')\} = 2(\tilde\sigma^{m_1 \ldots m_5} \sigma^{n_1 \ldots n_5})\delta(\sigma - \sigma'), \tag{35}
$$

with all other PB vanishing in the strong sense, we can introduce the second stage or twistor DB that in the subspace of the phase space defined by the projections of constraints (17),

$$\Phi^{+2}(\sigma) = 2v_{\dot{A}}^{\alpha+} p_{(\mu)\alpha\dot{A}}^{+} + \frac{1}{\alpha'}\left(\rho^{\tau+2} - is\varphi_{\sigma}^{+2}\right) \approx 0,$$

$$\Phi^{-2}(\sigma) = 2v_{A}^{\alpha-} p_{(\mu)\alpha A}^{-} + \frac{1}{\alpha'}\left(\rho^{\tau-2} - is\varphi_{\sigma}^{-2}\right) \approx 0, \tag{36}$$

$$\Phi^{I}(\sigma) = -v_{A}^{\alpha-}\gamma_{A\dot{A}}^{I} p_{(\mu)\alpha\dot{A}}^{+} - v_{\dot{A}}^{\alpha+}\tilde{\gamma}_{\dot{A}A}^{I} p_{(\mu)\alpha A}^{-} - \frac{is}{\alpha'}\varphi_{\sigma}^{I} \approx 0,$$

that complement those of (31) as well as $SO(1, 9)$ generators (26)–(29) and primary constraints (18), (22) and (23) coincide with the PB.

Therefore, we arrive at the following expression for the total Hamiltonian density,

$$\begin{aligned}
H_t(\tau, \sigma) = {} & \frac{\rho^{\sigma+2}}{2\alpha'}\left(\omega_{\sigma}^{-2} + \rho^{\tau-2}\right) + \frac{\rho^{\sigma-2}}{2\alpha'}\left(\omega_{\sigma}^{+2} - \rho^{\tau+2}\right) + a^{+2}\Phi^{-2} + a^{-2}\Phi^{+2} + a^{I}\Phi^{I} \\
& + l^{+2-2}M^{+2-2} + l^{+2I}M^{-2I} + l^{-2I}M^{+2I} + l^{IJ}M^{IJ} \\
& + b^{\mu+2}P_{\mu}^{-2} + b^{\mu-2}P_{\mu}^{+2} + \xi_{A}^{+}D_{A}^{-} + \xi_{\dot{A}}^{-}D_{\dot{A}}^{+},
\end{aligned} \tag{37}$$

as the linear combination of the remaining primary constraints with the bosonic $a(\sigma)$, $b(\sigma)$, $l(\sigma)$ and fermionic $\xi(\sigma)$ Lagrange multipliers to be determined from the consistency requirement. In the canonical formalism, evolution of any function of the phase-space variables is defined by its PB with the Hamiltonian

$$\dot{f}(Q, P) = \{f, \mathcal{H}\}, \tag{38}$$

where $\mathcal{H} = \int d\sigma\, H_t(\tau, \sigma)$. Following the Dirac method, the consistency requirement for the constraints is that they be weakly conserved, i.e. their PB with the Hamiltonian $\mathcal{H}$ weakly vanish[5]. In the present case, we find that the conservation of constraints $P_{\sigma}^{\pm 2} \approx 0$ yields the pair of secondary constraints

$$\omega_{\sigma}^{\pm 2} \mp \rho^{\tau\pm 2} \approx 0, \tag{39}$$

whereas from the conservation conditions for $P_{\tau}^{\pm 2} \approx 0$ we obtain the following equations for the Lagrange multipliers:

$$a^{+2} = -\frac{1}{2}\rho^{\sigma+2} + \frac{i}{16}\xi_{A}^{+}\eta_{A}^{+}, \qquad a^{-2} = \frac{1}{2}\rho^{\sigma-2} + \frac{i}{16}\xi_{\dot{A}}^{-}\eta_{\dot{A}}^{-}. \tag{40}$$

PB of $M^{IJ} \approx 0$ with $\mathcal{H}$ weakly vanish, while of $M^{+2-2} \approx 0$ with $\mathcal{H}$ weakly vanish provided one uses (40). Conservation of the $SO(1, 9)/(SO(1, 1) \times SO(8))$ coset generators $M^{\pm 2I} \approx 0$ results in the secondary constraints

$$\omega_{\sigma}^{I} \approx 0 \tag{41}$$

and the restriction for Lagrange multipliers a^{I}:

$$a^{I} = \frac{i}{16}(\eta^{+}\gamma^{I}\xi^{-} - \xi^{+}\gamma^{I}\eta^{-}). \tag{42}$$

Evaluating PB of $\Phi^{\pm 2} \approx 0$ and $\mathcal{H}$ one arrives at the equations

$$b^{\tau+2} = -\partial_{\sigma}\rho^{\sigma+2} - \frac{1}{2}\Omega_{\sigma}^{+2-2}\rho^{\sigma+2} + 2l^{+2-2}\rho^{\tau+2} + \frac{is}{4}\mathcal{D}_{\sigma}\eta_{A}^{+}\xi_{A}^{+}, \tag{43}$$

$$b^{\tau-2} = -\partial_{\sigma}\rho^{\sigma-2} + \frac{1}{2}\Omega_{\sigma}^{+2-2}\rho^{\sigma-2} - 2l^{+2-2}\rho^{\tau-2} + \frac{is}{4}\mathcal{D}_{\sigma}\eta_{\dot{A}}^{-}\xi_{\dot{A}}^{-}, \tag{44}$$

---

[5] PB of the primary and secondary constraints are given in appendix A.

while from the conservation conditions for $\Phi^I \approx 0$ and $\omega_\sigma^I \approx 0$ there stem the equations

$$\rho^{\tau-2}l^{+2I} + \rho^{\tau+2}l^{-2I} = \frac{1}{2}\left(\rho^{\sigma-2}\Omega_\sigma^{+2I} + \rho^{\sigma+2}\Omega_\sigma^{-2I}\right) - \frac{is}{8}(\xi^+\gamma^I \mathcal{D}_\sigma \eta^- - \mathcal{D}_\sigma \eta^+ \gamma^I \xi^-), \qquad (45)$$

$$\rho^{\tau-2}l^{+2I} - \rho^{\tau+2}l^{-2I} = \frac{1}{2}\left(\rho^{\sigma-2}\Omega_\sigma^{+2I} - \rho^{\sigma+2}\Omega_\sigma^{-2I}\right) - \frac{i}{8}(\xi^+\gamma^I \mathcal{D}_\sigma \eta^- - \mathcal{D}_\sigma \eta^+ \gamma^I \xi^-). \qquad (46)$$

Conservation of the secondary constraint $\omega^{+2} - \rho^{\tau+2} \approx 0$ gives another equation for $b^{\tau+2}$:

$$b^{\tau+2} = -\partial_\sigma \rho^{\sigma+2} - \frac{1}{2}\Omega_\sigma^{+2-2}\rho^{\sigma+2} + 2l^{+2-2}\rho^{\tau+2} + \frac{i}{4}\xi_A^+ \mathcal{D}_\sigma \eta_A^+. \qquad (47)$$

Its comparison with (43) reveals that either $s = -1$ or $\xi_A^+ \sim \mathcal{D}_\sigma \eta_A^+$. Analogously considering the conservation of the secondary constraint $\omega_\sigma^{-2} + \rho^{\tau-2} \approx 0$ we derive another equation for $b^{\tau-2}$:

$$b^{\tau-2} = -\partial_\sigma \rho^{\sigma-2} + \frac{1}{2}\Omega_\sigma^{+2-2}\rho^{\sigma-2} - 2l^{+2-2}\rho^{\tau-2} + \frac{i}{4}\mathcal{D}_\sigma \eta_{\dot{A}}^- \xi_{\dot{A}}^-. \qquad (48)$$

Its compatibility with (44) requires either $s = 1$ or $\xi_{\dot{A}}^- \sim \mathcal{D}_\sigma \eta_{\dot{A}}^-$. There remain to consider the consistency conditions for the fermionic constraints (18). For $D_A^- \approx 0$ we obtain $\xi_A^+ = -\frac{\rho^{\sigma-2}}{\rho^{\tau-2}}\mathcal{D}_\sigma \eta_A^+$ when $s = 1$, while in the case $s = -1$ no new restrictions on the Lagrange multipliers arise. The conservation condition for $D_{\dot{A}}^+ \approx 0$ yields $\xi_{\dot{A}}^- = -\frac{\rho^{\sigma+2}}{\rho^{\tau+2}}\mathcal{D}_\sigma \eta_{\dot{A}}^-$ when $s = -1$, but is trivial when $s = 1$. So we conclude that for $s = 1$ $\xi_A^+ = -\frac{\rho^{\sigma-2}}{\rho^{\tau-2}}\mathcal{D}_\sigma \eta_A^+$, while $\xi_{\dot{A}}^-$ remains undetermined, whereas when $s = -1$ $\xi_{\dot{A}}^- = -\frac{\rho^{\sigma+2}}{\rho^{\tau+2}}\mathcal{D}_\sigma \eta_{\dot{A}}^-$, but ξ_A^+ is free.

Upon substitution of the above-derived expressions for the Lagrange multipliers back into the Hamiltonian density (37) it turns into the following linear combination of the first-class constraints,

$$H_{t,s=1} = \rho^{\sigma+2}\Delta_{(-)}^{-2} + \rho^{\sigma-2}\widetilde{\Delta}_{(+)}^{+2} + l^{+2-2}\widetilde{M}^{+2-2} + l^{IJ}M^{IJ} + b^{\sigma+2}P_\sigma^{-2} + b^{\sigma-2}P_\sigma^{+2} + \xi_{\dot{A}}^- \widetilde{D}_{\dot{A}}^+ \approx 0, \qquad (49)$$

where

$$\Delta_{(-)}^{-2}(\sigma) = \frac{1}{2\alpha'}\left(\omega_\sigma^{-2} + \rho^{\tau-2}\right) - \frac{1}{2}\Phi^{-2} + \partial_\sigma P_\tau^{-2} - \frac{1}{2}\Omega_\sigma^{+2-2}P_\tau^{-2} + \frac{1}{2\rho^{\tau+2}}\Omega_\sigma^{-2I}M^{+2I} \approx 0, \qquad (50)$$

and

$$\widetilde{\Delta}_{(+)}^{+2}(\sigma) = \Delta_{(+)}^{+2} - \frac{1}{\rho^{\tau-2}}\mathcal{D}_\sigma \eta_A^+ \widetilde{D}_A^- \approx 0 \qquad (51)$$

are the generators of the reparametrizations[6]. In (51), we introduced the following combinations of the primary and secondary constraints:

$$\Delta_{(+)}^{+2}(\sigma) = \frac{1}{2\alpha'}\left(\omega_\sigma^{+2} - \rho^{\tau+2}\right) + \frac{1}{2}\Phi^{+2} + \partial_\sigma P_\tau^{+2} + \frac{1}{2}\Omega_\sigma^{+2-2}P_\tau^{+2} + \frac{1}{2\rho^{\tau-2}}\Omega_\sigma^{+2I}M^{-2I} \approx 0, \quad (52)$$

$$\widetilde{D}_A^-(\sigma) = D_A^- + \frac{i}{16}\eta_A^+ \Phi^{-2} - \frac{i}{16}\gamma_{A\dot{A}}^I \eta_{\dot{A}}^- \Phi^I - \frac{i}{8\rho^{\tau-2}}\gamma_{A\dot{A}}^I \mathcal{D}_\sigma \eta_{\dot{A}}^- M^{-2I} \approx 0. \qquad (53)$$

---

[6] In the above expressions and in what follows, lower $\pm$ indices in brackets of $\Delta_k^{\pm2}$ indicate the sign of the $\Phi^{\pm2} \approx 0$ constraint contribution and should not be confused with the $SO(1,1)$ indices.

Constraints ([52](#)) and ([50](#)) are the particular cases corresponding to $k = 1$ and $k = -1$, respectively, of the more general constraints

$$\Delta_k^{+2}(\sigma) = \frac{1}{2\alpha'}\left(\omega_\sigma^{+2} - \rho^{\tau+2}\right) + \frac{k}{2}\Phi^{+2} + \partial_\sigma P_\tau^{+2} + \frac{1}{2}\Omega_\sigma^{+2-2}P_\tau^{+2} + \frac{1}{2\rho^{\tau-2}}\Omega_\sigma^{+2I}M^{-2I} \approx 0, \quad (54)$$

and

$$\Delta_k^{-2}(\sigma) = \frac{1}{2\alpha'}\left(\omega_\sigma^{-2} + \rho^{\tau-2}\right) + \frac{k}{2}\Phi^{-2} + \partial_\sigma P_\tau^{-2} - \frac{1}{2}\Omega_\sigma^{+2-2}P_\tau^{-2} + \frac{1}{2\rho^{\tau+2}}\Omega_\sigma^{-2I}M^{+2I} \approx 0 \quad (55)$$

to be used below. Other bosonic first-class constraints

$$\widetilde{M}^{+2-2} = M^{+2-2} + 2\rho^{\tau+2}P_\tau^{-2} - 2\rho^{\tau-2}P_\tau^{+2} \approx 0 \tag{56}$$

and ([29](#)) generate $SO(1, 1) \times SO(8)$ gauge transformations. Eight fermionic first-class constraints

$$\widetilde{D}_{\dot{A}}^+ = D_{\dot{A}}^+ + \frac{i}{16}\eta_{\dot{A}}^-\Phi^{+2} - \frac{i}{16}\tilde{\gamma}_{\dot{A}A}^I\eta_A^+\Phi^I - \frac{i}{8\rho^{\tau-2}}\tilde{\gamma}_{\dot{A}A}^I\mathcal{D}_\sigma\eta_A^+M^{-2I} - \frac{i}{4}\mathcal{D}_\sigma\eta_{\dot{A}}^-P_\tau^{+2} \approx 0 \quad (57)$$

are responsible for the κ-symmetry.

When $s = -1$ we get

$$H_{t,s=-1} = \rho^{\sigma+2}\widetilde{\Delta}_{(-)}^{-2} + \rho^{\sigma-2}\Delta_{(+)}^{+2} + l^{+2-2}\widetilde{M}^{+2-2} + l^{IJ}M^{IJ} + b^{\sigma+2}P_\sigma^{-2} + b^{\sigma-2}P_\sigma^{+2} + \xi_{\dot{A}}^+\widetilde{D}_{\dot{A}}^{\prime-} \approx 0, \tag{58}$$

where

$$\widetilde{\Delta}_{(-)}^{-2} = \Delta_{(-)}^{-2} - \frac{1}{\rho^{\tau+2}}\mathcal{D}_\sigma\eta_{\dot{A}}^-\widetilde{D}_{\dot{A}}^{\prime+} \approx 0. \tag{59}$$

In ([59](#)) the second-class constraints $\widetilde{D}_{\dot{A}}^{\prime+} \approx 0$ are defined as

$$\widetilde{D}_{\dot{A}}^{\prime+} = D_{\dot{A}}^+ + \frac{i}{16}\eta_{\dot{A}}^-\Phi^{+2} - \frac{i}{16}\tilde{\gamma}_{\dot{A}A}^I\eta_A^+\Phi^I + \frac{i}{8\rho^{\tau+2}}\tilde{\gamma}_{\dot{A}A}^I\mathcal{D}_\sigma\eta_A^+M^{+2I} \approx 0, \quad (60)$$

whereas the generators of the κ-symmetry equal

$$\widetilde{D}_A^{\prime-}(\sigma) = D_A^- + \frac{i}{16}\eta_A^+\Phi^{-2} - \frac{i}{16}\gamma_{A\dot{A}}^I\eta_{\dot{A}}^-\Phi^I + \frac{i}{8\rho^{\tau+2}}\gamma_{A\dot{A}}^I\mathcal{D}_\sigma\eta_{\dot{A}}^-M^{+2I} + \frac{i}{4}\mathcal{D}_\sigma\eta_A^+P_\tau^{-2} \approx 0. \quad (61)$$

In what follows, for definiteness we concentrate on exploring the $s = 1$ case.

Now consider the canonical form of the irreducible κ-symmetry transformations generated on PB by the fermionic first-class constraints ([57](#)) according to the rule

$$\delta_\kappa f(\tau, \sigma) = \left\{\int d\sigma' \kappa_{\dot{B}}^-(\sigma')\widetilde{D}_{\dot{B}}^+(\sigma'), f\right\}. \tag{62}$$

A straightforward calculation yields that the supertwistor components transform as

$$\delta_\kappa \mu_{\dot{A}}^{\alpha-} = \frac{i}{2}\left(\kappa_{\dot{A}}^-\eta_A^+ + \frac{1}{8}(\kappa^-\tilde{\gamma}^I\eta^+)\tilde{\gamma}_{\dot{A}A}^I\right)v_A^{\alpha-} + \frac{i}{2}\left(\kappa_{\dot{A}}^-\eta_{\dot{B}}^- + \eta_{\dot{A}}^-\kappa_{\dot{B}}^- + \frac{1}{4}\delta_{\dot{A}\dot{B}}(\kappa^-\eta^-)\right)v_{\dot{B}}^{\alpha+}, \quad (63)$$

$$\delta_\kappa v_{\alpha\dot{A}}^- = 0, \qquad \delta_\kappa \eta_{\dot{A}}^- = \kappa_{\dot{A}}^-, \tag{64}$$

$$\delta_\kappa \mu_A^{\alpha+} = \frac{i}{8\rho^{\tau-2}}(\kappa^-\tilde{\gamma}^I\mathcal{D}_\sigma\eta^+)\gamma_{A\dot{A}}^I\mu_{\dot{A}}^{\alpha-} + \frac{i}{16}(\kappa^-\tilde{\gamma}^I\eta^+)\gamma_{A\dot{A}}^Iv_{\dot{A}}^{\alpha+} + \frac{i}{2}\eta_A^+\kappa_{\dot{B}}^-v_{\dot{B}}^{\alpha+}, \tag{65}$$

$$\delta_\kappa v_{\alpha A}^+(\eta_A^+) = \frac{i}{8\rho^{\tau-2}}(\kappa^-\tilde{\gamma}^I\mathcal{D}_\sigma\eta^+)\gamma_{A\dot{A}}^Iv_{\alpha\dot{A}}^-(\eta_{\dot{A}}^-). \tag{66}$$

We note that the 2D covariant form of the κ-symmetry transformations derived in the framework of the Lagrangian approach [51] reduces to the above expressions, provided one replaces all the τ-derivatives of the coordinates using their equations of motion.

3. The second-class constraints and Dirac brackets

The twistor realization of the 33 bosonic and 8 fermionic first-class constraints by which the $D = 10$ superstring in the twistor formulation is characterized has been exhibited above. The remaining primary and secondary constraints are of the second class. They can be classified according to their grading and the $SO(8)$ representation. Four vector constraints are represented by (27), (28) and

$$\Delta_k^I(\sigma) = \frac{1}{\alpha'}\omega_\sigma^I + k\Phi^I - \Omega_\sigma^{+2I} P_\tau^{-2} - \Omega_\sigma^{-2I} P_\tau^{+2} - \frac{1}{2}\mathscr{D}_\sigma^{IJ}\left(\frac{M^{+2J}}{\rho^{\tau+2}}\right) - \frac{1}{2}\mathscr{D}_\sigma^{IJ}\left(\frac{M^{-2J}}{\rho^{\tau-2}}\right) \approx 0,$$
(67)

where $\mathscr{D}_\sigma^{IJ} = \delta^{IJ}\partial_\sigma - \Omega_\sigma^{IJ}$ is the worldsheet-projected $SO(8)$ covariant differential. Four scalar constraints can be chosen as (22) and $\Delta_{(-)}^{+2} \approx 0$, $\Delta_{(+)}^{-2} \approx 0$ defined in (54) and (55). Finally, there are eight fermionic second-class constraints (53).

In the canonical approach, one of the possible options to take into account the second-class constraints is to introduce DB

$$\{f(\sigma), g(\sigma')\}_{\text{DB}} = \{f(\sigma), g(\sigma')\} - \{f(\sigma), \chi_{\mathsf{m}}\}\mathbf{C}^{-1\mathsf{mn}}\{\chi_{\mathsf{n}}, g(\sigma')\},$$
(68)

where χ_{m} denotes the set of the second-class constraints and $\mathbf{C}^{-1\mathsf{mn}}$ is the inverse of the Dirac matrix

$$\mathbf{C}_{\mathsf{mn}}(\sigma, \sigma') = \{\chi_{\mathsf{m}}(\sigma), \chi_{\mathsf{n}}(\sigma')\}.$$
(69)

For the above choice of the second-class constraints set, the Dirac matrix acquires the form

$$\mathbf{C}_{\mathsf{mn}} = J_{\mathsf{mn}} + \Lambda_{\mathsf{mn}},$$
(70)

where J_{mn} is the block-diagonal graded antisymmetric matrix and Λ_{mn} depends linearly on the constraints[7]. Explicitly J_{mn} reads

$$\alpha' J = \begin{array}{c} \\ M^{+2I} \\ \Delta_{(-)}^I \\ M^{-2I} \\ \Delta_{(+)}^I \\ \Delta_{(-)}^{+2} \\ P_\tau^{-2} \\ \Delta_{(+)}^{-2} \\ P_\tau^{+2} \\ \widetilde{D}_A^- \end{array} \begin{array}{|cccccccc|} \hline 0 & -2\rho^{\tau+2}\delta^{IJ} & & & & & & \\ 2\rho^{\tau+2}\delta^{IJ} & 0 & & & & & & \\ & & 0 & 2\rho^{\tau-2}\delta^{IJ} & & & \mathbf{0} & \\ & & -2\rho^{\tau-2}\delta^{IJ} & 0 & & & & \\ & & & & 0 & 1 & & \\ & & & & -1 & 0 & & \\ & & \mathbf{0} & & & & 0 & -1 \\ & & & & & & 1 & 0 \\ & & & & & & & \frac{\mathrm{i}\rho^{\tau-2}}{4}\delta_{AB} \\ \hline \end{array} \delta(\sigma - \sigma').$$
(71)

Then the inverse to the Dirac matrix (70) is given by

$$\mathbf{C}^{-1} = (I + J^{-1}\Lambda)J^{-1}$$
(72)

and can be expanded as the series

$$\mathbf{C}^{-1} = J^{-1} - J^{-1}\Lambda J^{-1} + J^{-1}\Lambda J^{-1}\Lambda J^{-1} - J^{-1}\Lambda J^{-1}\Lambda J^{-1}\Lambda J^{-1} + \cdots.$$
(73)

Since J is proportional to $(\alpha')^{-1}$ and its inverse depends on α' the inverse Dirac matrix is presented as the series in α'.[8] Expansion (73) suggests that we can evaluate $\mathbf{C}^{-1}$ perturbatively

---

[7] Expressions for the PB of the second-class constraints are given in appendix B.
[8] Note, however, that some entries of Λ have implicit dependence on $(\alpha')^{-1}$ through constraints (54), (55) and (67).

with the leading contribution determined by J^{-1} and the DB acquire the form

$$
\{f(\sigma), g(\sigma')\}_{\mathrm{DB}} = \{f(\sigma), g(\sigma')\} - 4\mathrm{i}\alpha' \int \frac{\mathrm{d}\sigma''}{\rho^{\tau-2}(\sigma'')} \{f(\sigma), \widetilde{D}_A^-(\sigma'')\}\{\widetilde{D}_A^-(\sigma''), g(\sigma')\}
$$

$$
- \frac{\alpha'}{2} \int \frac{\mathrm{d}\sigma''}{\rho^{\tau+2}(\sigma'')} (\{f(\sigma), M^{+2I}(\sigma'')\}\{\Delta_{(-)}^I(\sigma''), g(\sigma')\}
$$

$$
- \{f(\sigma), \Delta_{(-)}^I(\sigma'')\}\{M^{+2I}(\sigma''), g(\sigma')\})
$$

$$
+ \frac{\alpha'}{2} \int \frac{\mathrm{d}\sigma''}{\rho^{\tau-2}(\sigma'')} (\{f(\sigma), M^{-2I}(\sigma'')\}\{\Delta_{(+)}^I(\sigma''), g(\sigma')\}
$$

$$
- \{f(\sigma), \Delta_{(+)}^I(\sigma'')\}\{M^{-2I}(\sigma''), g(\sigma')\})
$$

$$
+ \alpha' \int \mathrm{d}\sigma'' (\{f(\sigma), \Delta_{(-)}^{+2}(\sigma'')\}\{P_\tau^{-2}(\sigma''), g(\sigma')\}
$$

$$
- \{f(\sigma), P_\tau^{-2}(\sigma'')\}\{\Delta_{(-)}^{+2}(\sigma''), g(\sigma')\})
$$

$$
- \alpha' \int \mathrm{d}\sigma'' (\{f(\sigma), \Delta_{(+)}^{-2}(\sigma'')\}\{P_\tau^{+2}(\sigma''), g(\sigma')\}
$$

$$
- \{f(\sigma), P_\tau^{+2}(\sigma'')\}\{\Delta_{(+)}^{-2}(\sigma''), g(\sigma')\}) + O(J^{-2}). \tag{74}
$$

Expressions (68) and (74) can be used to evaluate the DB relations of the superstring phase-space variables. So for the two sets of supertwistors (2) associated with the worldsheet light-like directions, we get

$$
\{Z_A^{\Lambda+}(\sigma), Z_B^{\Sigma+}(\sigma')\}_{\mathrm{DB}} = \frac{4\mathrm{i}\alpha'}{\rho^{\tau-2}} D_{AC}^\Lambda D_{BC}^\Sigma \delta(\sigma - \sigma')
$$

$$
+ \frac{\alpha'}{2\rho^{\tau-2}} \gamma_{A\dot{A}}^I \gamma_{B\dot{B}}^I (V_{\dot{A}}^{\Lambda+} Z_{\dot{B}}^{\Sigma-} - Z_{\dot{A}}^{\Lambda-} V_{\dot{B}}^{\Sigma+}) \delta(\sigma - \sigma')
$$

$$
+ \frac{\alpha'}{2\rho^{\tau-2}} \gamma_{A\dot{A}}^I Z_{\dot{A}}^{\Lambda-}(\sigma) \gamma_{B\dot{B}}^J \mathscr{D}_\sigma^{IJ} \left(\frac{1}{\rho^{\tau-2}} Z_{\dot{B}}^{\Sigma-}(\sigma) \delta(\sigma - \sigma') \right) + O(\alpha'^2), \tag{75}
$$

$$
\{Z_{\dot{A}}^{\Lambda-}(\sigma), Z_{\dot{B}}^{\Sigma-}(\sigma')\}_{\mathrm{DB}} = \frac{4\mathrm{i}\alpha'}{\rho^{\tau-2}} D_{\dot{A}C}^{\Lambda-2} D_{\dot{B}C}^{\Sigma-2} \delta(\sigma - \sigma')
$$

$$
+ \frac{\alpha'}{2\rho^{\tau+2}} \tilde{\gamma}_{\dot{A}A}^I \tilde{\gamma}_{\dot{B}B}^I (V_A^{\Lambda-} Z_B^{\Sigma+} - Z_A^{\Lambda+} V_B^{\Sigma-}) \delta(\sigma - \sigma')
$$

$$
- \frac{\alpha'}{2\rho^{\tau+2}} \tilde{\gamma}_{\dot{A}A}^I Z_A^{\Lambda+}(\sigma) \tilde{\gamma}_{\dot{B}B}^J \mathscr{D}_\sigma^{IJ} \left(\frac{1}{\rho^{\tau+2}} Z_B^{\Sigma+}(\sigma) \delta(\sigma - \sigma') \right) + O(\alpha'^2), \tag{76}
$$

as well as

$$
\{Z_A^{\Lambda+}(\sigma), Z_{\dot{B}}^{\Sigma-}(\sigma')\}_{\mathrm{DB}} = \frac{4\mathrm{i}\alpha'}{\rho^{\tau-2}} D_{AC}^\Lambda D_{\dot{B}C}^{\Sigma-2} \delta(\sigma - \sigma')
$$

$$
+ \frac{\alpha'}{2} \gamma_{A\dot{A}}^I \tilde{\gamma}_{\dot{B}B}^I \left(\frac{1}{\rho^{\tau+2}} V_{\dot{A}}^{\Lambda+} Z_B^{\Sigma+} - \frac{1}{\rho^{\tau-2}} Z_{\dot{A}}^{\Lambda-} V_B^{\Sigma-} \right) \delta(\sigma - \sigma') + O(\alpha'^2), \tag{77}
$$

where the inverse spinor harmonic matrix components have been promoted to supertwistors $V_A^{\Lambda-} = (v_A^{\alpha-}, 0, 0)$, $V_{\dot{A}}^{\Lambda+} = (v_{\dot{A}}^{\alpha+}, 0, 0)$ and the following quantities have been introduced:

$$
\{\widetilde{D}_A^-(\sigma), Z_B^{\Lambda+}(\sigma')\} = D_{AB}^\Lambda \delta(\sigma - \sigma'),
$$

$$
D_{AB}^\Lambda = -D_{BA}^\Lambda = \frac{\mathrm{i}}{8\rho^{\tau-2}} \gamma_{A\dot{A}}^I \mathcal{D}_\sigma \eta_{\dot{A}}^- \gamma_{B\dot{B}}^I Z_{\dot{B}}^{\Lambda-} + \frac{\mathrm{i}}{2} \left(\delta_{AB} \eta_{\dot{B}}^- + \frac{1}{8} \gamma_{A\dot{A}}^I \eta_{\dot{A}}^- \gamma_{B\dot{B}}^I \right) V_{\dot{B}}^{\Lambda+}
$$

$$
+ \frac{\mathrm{i}}{2} \left(\delta_{AB} \eta_C^+ - \delta_{AC} \eta_B^+ + \frac{1}{8} \delta_{BC} \eta_A^+ \right) V_C^{\Lambda-} + \delta_{AB} J^\Lambda, \qquad J^\Lambda = (0, 0, 1), \tag{78}
$$

$$\left\{\tilde{D}_A^-(\sigma),\, Z_{\dot{B}}^{\Lambda-}(\sigma')\right\} = D_{A\dot{B}}^{\Lambda-2}\delta(\sigma - \sigma'),$$

$$D_{A\dot{B}}^{\Lambda-2} = -D_{\dot{B}A}^{\Lambda-2} = -\frac{\mathrm{i}}{2}\left(\delta_{AB}\eta_{\dot{B}}^- - \frac{1}{8}\gamma_{A\dot{A}}^I\eta_{\dot{A}}^-\tilde{\gamma}_{\dot{B}B}^I\right)V_B^{\Lambda-}. \tag{79}$$

To the first order in α' supertwistors also satisfy nonzero DB relations with the worldsheet densities

$$\left\{Z_A^{\Lambda+}(\sigma),\, \rho^{\tau-2}(\sigma')\right\}_{\mathrm{DB}} = \alpha' V_A^{\Lambda-}\delta(\sigma - \sigma') - \frac{\alpha'}{2\rho^{\tau-2}}\Omega_\sigma^{-2I}\gamma_{A\dot{A}}^I Z_{\dot{A}}^{\Lambda-}\delta(\sigma - \sigma') + O(\alpha'^2),$$

$$\left\{Z_{\dot{A}}^{\Lambda-}(\sigma),\, \rho^{\tau+2}(\sigma')\right\}_{\mathrm{DB}} = \alpha' V_{\dot{A}}^{\Lambda+}\delta(\sigma - \sigma') + \frac{\alpha'}{2\rho^{\tau+2}}\Omega_\sigma^{+2I}\tilde{\gamma}_{\dot{A}A}^I Z_A^{\Lambda+}\delta(\sigma - \sigma') + O(\alpha'^2). \tag{80}$$

DB deformation of PB relations for the phase-space variables results in the deformation of the first-class constraint algebra. For the κ-symmetry generators, (57) one obtains

$$\left\{\widetilde{D}_{\dot{A}}^+(\sigma),\, \widetilde{D}_{\dot{B}}^+(\sigma')\right\}_{\mathrm{DB}} = \frac{\mathrm{i}}{4}\delta_{\dot{A}\dot{B}}\widetilde{\Delta}_{(+)}^{+2}\delta(\sigma - \sigma')$$

$$- \frac{\alpha'}{16\rho^{\tau+2}(\rho^{\tau-2})^2}\tilde{\gamma}_{\dot{A}A}^I\mathcal{D}_\sigma\eta_A^+\tilde{\gamma}_{\dot{B}B}^J\mathcal{D}_\sigma\eta_B^+ M^{IJ}\Delta_{(-)}^{-2}\delta(\sigma - \sigma')$$

$$+ \frac{\alpha'}{2}\Gamma_{\dot{A}}^{+I}(\sigma)\mathscr{D}_\sigma^{IJ}\Gamma_{\dot{B}}^{+J}(\sigma)\delta(\sigma - \sigma') + O(J^{-2}), \tag{81}$$

where $\Gamma_{\dot{A}}^{+I} = -\frac{1}{4\rho^{\tau+2}\rho^{\tau-2}}\tilde{\gamma}_{\dot{A}A}^J\mathcal{D}_\sigma\eta_A^+\left(\frac{1}{2}\delta^{JI}\widetilde{M}^{+2-2}+M^{JI}\right)$. The first term on the rhs proportional to the reparametrization generator (51) is the PB contribution, while the terms containing products of $SO(1,1)$, $SO(8)$ generators and the reparametrization generator (50) correspond to the deformation. The DB of the κ-symmetry and corresponding reparametrization generators have the form

$$\left\{\widetilde{\Delta}_{(+)}^{+2}(\sigma),\, \widetilde{D}_{\dot{A}}^+(\sigma')\right\}_{\mathrm{DB}} = \frac{\mathrm{i}\alpha'}{4\rho^{\tau-2}}\tilde{\gamma}_{\dot{A}A}^I\mathcal{D}_\sigma\eta_A^+\left(A^{+2I} - \frac{1}{2}\mathcal{D}_\sigma\eta^+\gamma^I\widetilde{D}^+\right)\Delta_{(-)}^{-2}\delta(\sigma - \sigma')$$

$$+ \frac{\mathrm{i}\alpha'}{2}A^{+2I}(\sigma)\mathscr{D}_\sigma^{IJ}\Gamma_{\dot{A}}^{+J}(\sigma)\delta(\sigma - \sigma') + O(J^{-2}), \tag{82}$$

where

$$A^{+2I} = \frac{1}{\rho^{\tau+2}\rho^{\tau-2}}\left[(\mathcal{D}_\sigma\eta^+\gamma^I\widetilde{D}^+) + \left(-\frac{1}{2}\delta^{IJ}\widetilde{M}^{+2-2} + M^{IJ}\right)\left(\Omega_\sigma^{+2J} + \frac{\mathrm{i}}{4\rho^{\tau-2}}(\mathcal{D}_\sigma\eta^+\gamma^J\mathcal{D}_\sigma\eta^-)\right)\right]. \tag{83}$$

Observe that equal-to-zero PB contribution becomes supplemented by the quadratic terms in the first-class constraints. At the same time at the lowest order in J^{-1} DB of the κ-symmetry generators and the reparametrization generator (50) coincide with the PB

$$\left\{\Delta_{(-)}^{-2}(\sigma),\, \widetilde{D}_{\dot{A}}^+(\sigma')\right\}_{\mathrm{DB}} = \frac{\mathrm{i}}{2}\Gamma_{\dot{A}}^{+I}\Omega_\sigma^{-2I}\delta(\sigma - \sigma') + O(J^{-2})$$

$$= -\frac{\mathrm{i}}{8}\tilde{\gamma}_{\dot{A}A}^I\mathcal{D}_\sigma\eta_A^+ B^{-2I}\delta(\sigma - \sigma') + O(J^{-2}), \tag{84}$$

where $B^{-2I} = \frac{1}{\rho^{\tau+2}\rho^{\tau-2}}\left(\frac{1}{2}\delta^{IJ}\widetilde{M}^{+2-2} + M^{IJ}\right)\Omega_\sigma^{-2J}$. Equal to zero diagonal PB relations of the reparametrization generators on transition to DB receive contributions quadratic in $SO(1,1)$ and $SO(8)$ generators

$$\left\{\widetilde{\Delta}_{(+)}^{+2}(\sigma),\, \widetilde{\Delta}_{(+)}^{+2}(\sigma')\right\}_{\mathrm{DB}} = -\frac{\alpha'}{2}A^{+2I}(\sigma)\mathscr{D}_\sigma^{IJ}A^{+2J}(\sigma)\delta(\sigma - \sigma') + O(J^{-2}), \tag{85}$$

$$\left\{\Delta_{(-)}^{-2}(\sigma),\, \Delta_{(-)}^{-2}(\sigma')\right\}_{\mathrm{DB}} = \frac{\alpha'}{2}B^{-2I}(\sigma)\mathscr{D}_\sigma^{IJ}B^{-2J}(\sigma)\delta(\sigma - \sigma') + O(J^{-2}), \tag{86}$$

while that of different reparametrization generators at the lowest order in J^{-1} become deformed by the terms proportional to the product of the reparametrization generator (50) with the generators of the κ-symmetry and $SO(1,1)$, $SO(8)$ generators

$$
\begin{aligned}
\left\{\widetilde{\Delta}^{+2}_{(+)}(\sigma), \Delta^{-2}_{(-)}(\sigma')\right\}_{\mathrm{DB}} = {} & \frac{1}{2\rho^{\tau+2}\rho^{\tau-2}}(\mathcal{D}_\sigma\eta^+\gamma^I\widetilde{D}^+)\Omega_\sigma^{-2I}\left(1 + \frac{\alpha'}{\rho^{\tau-2}}\Delta^{-2}_{(-)}\right)\delta(\sigma-\sigma') \\
& - \frac{1}{2}\left[\Omega_\sigma^{+2I}\left(1 + \frac{\alpha'}{\rho^{\tau-2}}\Delta^{-2}_{(-)}\right) + \frac{\mathrm{i}}{4\rho^{\tau-2}}(\mathcal{D}_\sigma\eta^+\gamma^I\mathcal{D}_\sigma\eta^-)\left(1 + \frac{2\alpha'}{\rho^{\tau-2}}\Delta^{-2}_{(-)}\right)\right] \\
& \times B^{-2I}\delta(\sigma-\sigma') + O(J^{-2}).
\end{aligned} \tag{87}
$$

PB of the $SO(1,1)$ and $SO(8)$ generators (56) and (29) and the second-class constraints are determined by their properties under the $SO(1,1) \times SO(8)$ transformations and hence are proportional to the second-class constraints. So the DB involving $\widetilde{M}^{+2-2} \approx 0$ and/or $M^{IJ} \approx 0$ are equal to the corresponding PB.

4. Conclusion

The present paper investigates the canonical approach application to the $D = 10$ superstring first-order action involving spinor harmonics and formulated in terms of supertwistor variables generalizing Penrose–Ferber ones. We have identified the primary and secondary constraints on the supertwistors and conjugate momenta, analyzed their consistency, and as a result obtained the set of first-class constraints that includes twistor realizations of the reparametrization, $SO(1,1) \times SO(8)$ gauge symmetry and κ-symmetry generators. The superstring model is also characterized by the second-class constraints that can be taken into account by constructing DB. To this end, we have chosen the basis in the space of the second-class constraints such that the Dirac matrix acquires the form of the sum of block-diagonal graded antisymmetric matrix J proportional to $(\alpha')^{-1}$ and the one linear in the constraints. So the DB can be evaluated perturbatively as the series in J^{-1}. Introduction of DB leads to the deformation of the first-class constraint algebra; the deformation parameter can be identified with α'. We have explicitly found the DB deformation of the first-class constraint algebra up to quadratic terms in the constraints, although it can be calculated to any order in J^{-1}. One could expect some simplification of the expression for DB by choosing such representation for the second-class constraints for which the Dirac matrix becomes equal to J. This, however, requires an addition to the obtained second-class constraints of the terms containing higher powers of the constraints to compensate weakly vanishing contributions to their PB. Another way to handle the second-class constraints is to bring them by canonical transformation to the special form[9] and then solve with respect to the subset of the canonically conjugate variables equal in number to the second-class constraints [64]. An alternative mode could be to consider the covariant supertwistor analog of the semi-lightcone gauge approach to the GS superstring quantization [65]. Its examination for the superstring in the twistor formulation has been initiated in [66] on the example of the $D = 4$ model. All these possibilities are under consideration.

Acknowledgment

---

[9] Note that the conjugate pairs of the second-class constraints $(P_\tau^{\pm 2}, \Delta^{\mp 2}_{(\pm)})$ are already brought to the special form.

Appendix A. PB relations of the primary and secondary constraints

Fermionic constraints (18) satisfy the following nonzero PB relations between themselves,

$$
\{D_A^-(\sigma), D_B^-(\sigma')\} = \frac{1}{8\alpha'}\delta_{AB}\big(\mathrm{i}s\omega_\sigma^{-2} - \mathrm{i}\rho^{\tau-2} - s\varphi_\sigma^{-2}\big)\delta(\sigma - \sigma'),
$$

$$
\{D_{\dot A}^+(\sigma), D_{\dot B}^+(\sigma')\} = \frac{1}{8\alpha'}\delta_{\dot A\dot B}\big(\mathrm{i}s\omega_\sigma^{+2} - \mathrm{i}\rho^{\tau+2} - s\varphi_\sigma^{+2}\big)\delta(\sigma - \sigma'), \qquad \text{(A.1)}
$$

$$
\{D_A^-(\sigma), D_{\dot B}^+(\sigma')\} = \frac{s}{8\alpha'}\gamma_{A\dot B}^I\big(\varphi_\sigma^I - \mathrm{i}\omega_\sigma^I\big)\delta(\sigma - \sigma')
$$

and with the Φ-constraints (17)

$$
\{D_A^-(\sigma), \Phi^{+2}(\sigma')\} = \frac{\mathrm{i}s}{4\alpha'}\mathcal{D}_\sigma\eta_A^+\delta(\sigma - \sigma'), \quad \{D_A^-(\sigma), \Phi^I(\sigma')\} = \frac{\mathrm{i}s}{8\alpha'}\gamma_{A\dot A}^I\mathcal{D}_\sigma\eta_{\dot A}^-\delta(\sigma - \sigma'),
$$

$$
\{D_{\dot A}^+(\sigma), \Phi^{-2}(\sigma')\} = \frac{\mathrm{i}s}{4\alpha'}\mathcal{D}_\sigma\eta_{\dot A}^-\delta(\sigma - \sigma'), \quad \{D_{\dot A}^+(\sigma), \Phi^I(\sigma')\} = \frac{\mathrm{i}s}{8\alpha'}\tilde\gamma_{\dot A A}^I\mathcal{D}_\sigma\eta_A^+\delta(\sigma - \sigma').
$$

$$\text{(A.2)}$$

σ-components of the worldsheet projections of ω 1-forms that enter the secondary constraints (39) and (41) satisfy the PB relations with the fermionic constraints

$$
\{D_A^-(\sigma), \omega_\sigma^{+2}(\sigma')\} = \frac{\mathrm{i}}{8}\eta_A^+(\sigma)\Big(\partial_\sigma - \frac{1}{2}\Omega_\sigma^{+2-2}\Big)\delta(\sigma - \sigma')
$$

$$
+ \frac{\mathrm{i}}{4}\Big(\mathcal{D}_\sigma\eta_A^+ + \frac{1}{4}\gamma_{A\dot A}^I\eta_{\dot A}^-\Omega_\sigma^{+2I}\Big)\delta(\sigma - \sigma'),
$$

$$
\{D_A^-(\sigma), \omega_\sigma^{-2}(\sigma')\} = \frac{\mathrm{i}}{16}\gamma_{A\dot A}^I\eta_{\dot A}^-\Omega_\sigma^{-2I}\delta(\sigma - \sigma'),
$$

$$
\{D_A^-(\sigma), \omega_\sigma^I(\sigma')\} = -\frac{\mathrm{i}}{16}\mathscr{D}_{\sigma'}^{IJ}\gamma_{A\dot A}^J\eta_{\dot A}^-(\sigma)\delta(\sigma - \sigma') + \frac{\mathrm{i}}{8}\Big(\gamma_{A\dot A}^I\mathcal{D}_\sigma\eta_{\dot A}^- + \frac{1}{2}\eta_A^+\Omega_\sigma^{-2I}\Big)\delta(\sigma - \sigma'),
$$

$$
\{D_{\dot A}^+(\sigma), \omega_\sigma^{-2}(\sigma')\} = \frac{\mathrm{i}}{8}\eta_{\dot A}^-(\sigma)\Big(\partial_\sigma + \frac{1}{2}\Omega_\sigma^{+2-2}\Big)\delta(\sigma - \sigma')
$$

$$
+ \frac{\mathrm{i}}{4}\Big(\mathcal{D}_\sigma\eta_{\dot A}^- + \frac{1}{4}\tilde\gamma_{\dot A A}^I\eta_A^+\Omega_\sigma^{-2I}\Big)\delta(\sigma - \sigma'),
$$

$$
\{D_{\dot A}^+(\sigma), \omega^{+2}(\sigma')\} = \frac{\mathrm{i}}{16}\tilde\gamma_{\dot A A}^I\eta_A^+\Omega_\sigma^{+2I}\delta(\sigma - \sigma'),
$$

$$
\{D_{\dot A}^+(\sigma), \omega_\sigma^I(\sigma')\} = -\frac{\mathrm{i}}{16}\mathscr{D}_{\sigma'}^{IJ}\tilde\gamma_{\dot A A}^J\eta_A^+(\sigma)\delta(\sigma - \sigma') + \frac{\mathrm{i}}{8}\Big(\tilde\gamma_{\dot A A}^I\mathcal{D}_\sigma\eta_A^+ + \frac{1}{2}\eta_{\dot A}^-\Omega_\sigma^{+2I}\Big)\delta(\sigma - \sigma'),
$$

$$\text{(A.3)}$$

and with the Φ-constraints

$$
\{\Phi^{\pm2}(\sigma), \omega_\sigma^{\mp2}(\sigma')\} = -2\big(\partial_\sigma \pm \tfrac{1}{2}\Omega_\sigma^{+2-2}\big)\delta(\sigma - \sigma'),
$$

$$
\{\Phi^I(\sigma), \omega_\sigma^{\pm2}(\sigma')\} = \{\omega_\sigma^I(\sigma), \Phi^{\pm2}(\sigma')\} = \Omega_\sigma^{\pm2I}\delta(\sigma - \sigma'). \qquad \text{(A.4)}
$$

The PB of $SO(1, 1)$ and $SO(1, 9)/(SO(1, 1) \times SO(8))$ coset generators (26)–(28) with the fermionic constraints equal

$$
\{M^{+2-2}(\sigma), D_A^-(\sigma')\} = -\Big(D_A^- + \frac{\mathrm{i}\rho^{\tau-2}}{8\alpha'}\eta_A^+\Big)\delta(\sigma - \sigma'),
$$

$$
\{M^{+2-2}(\sigma), D_{\dot A}^+(\sigma')\} = \Big(D_{\dot A}^+ + \frac{\mathrm{i}\rho^{\tau+2}}{8\alpha'}\eta_{\dot A}^-\Big)\delta(\sigma - \sigma'),
$$

$$\{M^{+2I}(\sigma), D_{\dot{A}}^{-}(\sigma')\} = \gamma_{A\dot{A}}^{I}\left(D_{\dot{A}}^{+} + \frac{\mathrm{i}\rho^{\tau+2}}{16\alpha'}\eta_{\dot{A}}^{-}\right)\delta(\sigma - \sigma'),$$

$$\{M^{+2I}(\sigma), D_{A}^{+}(\sigma')\} = \frac{\mathrm{i}\rho^{\tau+2}}{16\alpha'}\tilde{\gamma}_{\dot{A}A}^{I}\eta_{A}^{+}\delta(\sigma - \sigma'),$$

$$\{M^{-2I}(\sigma), D_{\dot{A}}^{-}(\sigma')\} = \frac{\mathrm{i}\rho^{\tau-2}}{16\alpha'}\gamma_{A\dot{A}}^{I}\eta_{\dot{A}}^{-}\delta(\sigma - \sigma'),$$

$$\{M^{-2I}(\sigma), D_{\dot{A}}^{+}(\sigma')\} = \tilde{\gamma}_{\dot{A}A}^{I}\left(D_{A}^{-} + \frac{\mathrm{i}\rho^{\tau-2}}{16\alpha'}\eta_{A}^{+}\right)\delta(\sigma - \sigma') \qquad (A.5)$$

while with the Φ-constraints read

$$\{M^{+2-2}(\sigma), \Phi^{\pm2}(\sigma')\} = \pm2\left(\Phi^{\pm2} - \frac{1}{\alpha'}\rho^{\tau\pm2}\right)\delta(\sigma - \sigma'),$$

$$\{M^{\pm2I}(\sigma), \Phi^{\mp2}(\sigma')\} = -2\Phi^{I}\delta(\sigma - \sigma'), \qquad (A.6)$$

$$\{M^{\pm2I}(\sigma), \Phi^{J}(\sigma')\} = -\delta^{IJ}\left(\Phi^{\pm2} - \frac{\rho^{\tau\pm2}}{\alpha'}\right)\delta(\sigma - \sigma').$$

Finally, the PB of the $SO(1,1)$ and $SO(1,9)/(SO(1,1) \times SO(8))$ generators and the σ-components of ω 1-forms are given by

$$\{M^{+2-2}(\sigma), \omega_{\sigma}^{\pm2}(\sigma')\} = \pm2\omega_{\sigma}^{\pm2}\delta(\sigma - \sigma'),$$

$$\{M^{\pm2I}(\sigma), \omega_{\sigma}^{\mp2}(\sigma')\} = -2\omega_{\sigma}^{I}\delta(\sigma - \sigma'), \qquad (A.7)$$

$$\{M^{\pm2I}(\sigma), \omega_{\sigma}^{J}(\sigma')\} = -\delta^{IJ}\omega^{\pm2}\delta(\sigma - \sigma').$$

Rhs of the PB of $SO(8)$ generators $M^{IJ} \approx 0$ with the fermionic constraints, Φ-constraints and the σ-components of ω 1-forms are defined by their transformation properties under the $SO(8)$ transformations and exhibit no 'anomalous' contributions as opposed to the above-given PB.

Appendix B. PB relations of the second-class constraints

PB of the $SO(8)$ vector constraints (27), (28) and (67) are given by

$$\{\Delta_{k}^{I}(\sigma), \Delta_{k'}^{J}(\sigma')\} = \left(\frac{\Delta_{k}^{-2}}{\rho^{\tau-2}}(\sigma) + \frac{\Delta_{k'}^{-2}}{\rho^{\tau-2}}(\sigma')\right)\mathscr{D}_{\sigma}^{IJ}\delta(\sigma - \sigma')$$

$$+ \left(\frac{\Delta_{k}^{+2}}{\rho^{\tau+2}}(\sigma) + \frac{\Delta_{k'}^{+2}}{\rho^{\tau+2}}(\sigma')\right)\mathscr{D}_{\sigma}^{IJ}\delta(\sigma - \sigma')$$

$$+ \frac{1}{4}\left(\frac{M^{+2K}}{\rho^{\tau+2}} - \frac{M^{-2K}}{\rho^{\tau-2}}\right)\left(\frac{\Omega_{\sigma}^{+2K}}{\rho^{\tau+2}} - \frac{\Omega_{\sigma}^{-2K}}{\rho^{\tau-2}}\right)(\sigma)\mathscr{D}_{\sigma}^{IJ}\delta(\sigma - \sigma')$$

$$+ \frac{1}{4}\left(\frac{M^{+2K}}{\rho^{\tau+2}} - \frac{M^{-2K}}{\rho^{\tau-2}}\right)\left(\frac{\Omega_{\sigma}^{+2K}}{\rho^{\tau+2}} - \frac{\Omega_{\sigma}^{-2K}}{\rho^{\tau-2}}\right)(\sigma')\mathscr{D}_{\sigma'}^{IJ}\delta(\sigma - \sigma')$$

$$+ \frac{1}{4}\left(\mathscr{D}_{\sigma}^{IK}\delta^{JL} - \mathscr{D}_{\sigma'}^{JK}\delta^{IL}\right)\left(\frac{M^{+2K}}{\rho^{\tau+2}} - \frac{M^{-2K}}{\rho^{\tau-2}}\right)\left(\frac{\Omega_{\sigma}^{+2L}}{\rho^{\tau+2}} - \frac{\Omega_{\sigma}^{-2L}}{\rho^{\tau-2}}\right)(\sigma)\delta(\sigma - \sigma')$$

$$- \mathscr{D}_{\sigma}^{IK}\mathscr{D}_{\sigma'}^{JL}\frac{M^{KL}}{\rho^{\tau+2}\rho^{\tau-2}}(\sigma)\delta(\sigma - \sigma'), \qquad (B.1)$$

$$\left\{M^{+2I}(\sigma), \Delta_k^J(\sigma')\right\} = \frac{(k-1)\rho^{\tau+2}}{\alpha'}\delta^{IJ}\delta(\sigma-\sigma') - 2\delta^{IJ}\Delta_k^{+2}\delta(\sigma-\sigma')$$
$$-\left[\frac{\widetilde{M}^{+2-2}}{2\rho^{\tau-2}} - \frac{(\rho^{\tau+2}P_\tau^{-2} + \rho^{\tau-2}P_\tau^{+2})}{\rho^{\tau-2}}\right](\sigma)\mathscr{D}_\sigma^{IJ}\delta(\sigma-\sigma')$$
$$+\mathscr{D}_{\sigma'}^{JK}\frac{M^{IK}}{\rho^{\tau-2}}(\sigma)\delta(\sigma-\sigma')$$
$$-\frac{1}{2}(\delta^{IJ}\delta^{KL} - \delta^{IK}\delta^{JL})\left(\frac{M^{+2K}}{\rho^{\tau+2}} - \frac{M^{-2K}}{\rho^{\tau-2}}\right)\Omega_\sigma^{+2L}\delta(\sigma-\sigma'), \qquad \text{(B.2)}$$

$$\left\{M^{-2I}(\sigma), \Delta_k^J(\sigma')\right\} = \frac{(k+1)\rho^{\tau-2}}{\alpha'}\delta^{IJ}\delta(\sigma-\sigma') - 2\delta^{IJ}\Delta_k^{-2}\delta(\sigma-\sigma')$$
$$-\left[\frac{\widetilde{M}^{+2-2}}{2\rho^{\tau+2}} + \frac{(\rho^{\tau+2}P_\tau^{-2} + \rho^{\tau-2}P_\tau^{+2})}{\rho^{\tau+2}}\right](\sigma)\mathscr{D}_\sigma^{IJ}\delta(\sigma-\sigma')$$
$$+\mathscr{D}_{\sigma'}^{JK}\frac{M^{IK}}{\rho^{\tau+2}}(\sigma)\delta(\sigma-\sigma')$$
$$+\frac{1}{2}(\delta^{IJ}\delta^{KL} - \delta^{IK}\delta^{JL})\left(\frac{M^{+2K}}{\rho^{\tau+2}} - \frac{M^{-2K}}{\rho^{\tau-2}}\right)\Omega_\sigma^{-2L}\delta(\sigma-\sigma'). \qquad \text{(B.3)}$$

PB of the constraints (67) and constraints (55), (54) and (22) equal

$$\left\{\Delta_k^I(\sigma), \Delta_{(+)}^{-2}(\sigma')\right\} = \Omega_\sigma^{-2I}\frac{\Delta_k^{+2}}{\rho^{\tau+2}}\delta(\sigma-\sigma')$$
$$+\frac{1}{2}\mathscr{D}_\sigma^{IJ}\left(\frac{1}{2}\delta^{JK}\widetilde{M}^{+2-2} + M^{JK}\right)\frac{\Omega_\sigma^{-2K}}{\rho^{\tau+2}\rho^{\tau-2}}\delta(\sigma-\sigma')$$
$$+\frac{1}{4\rho^{\tau+2}}\left(\Omega_\sigma^{+2I}\Omega_\sigma^{-2J} + \Omega_\sigma^{-2I}\Omega_\sigma^{+2J}\right)\left(\frac{M^{+2J}}{\rho^{\tau+2}} - \frac{M^{-2J}}{\rho^{\tau-2}}\right)\delta(\sigma-\sigma')$$
$$+\frac{1}{4}\mathscr{D}_\sigma^{IJ}\frac{1}{\rho^{\tau+2}}\left[2\Delta_{(+)}^J - \left(\mathscr{D}_\sigma^{JK}\left(\frac{M^{+2K}}{\rho^{\tau+2}} - \frac{M^{-2K}}{\rho^{\tau-2}}\right)\right)\right](\sigma)\delta(\sigma-\sigma'), \quad \text{(B.4)}$$

$$\left\{\Delta_k^I(\sigma), \Delta_{(-)}^{+2}(\sigma')\right\} = \Omega_\sigma^{+2I}\frac{\Delta_k^{-2}}{\rho^{\tau-2}}\delta(\sigma-\sigma')$$
$$+\frac{1}{2}\mathscr{D}_\sigma^{IJ}\left(-\frac{1}{2}\delta^{JK}\widetilde{M}^{+2-2} + M^{JK}\right)\frac{\Omega_\sigma^{+2K}}{\rho^{\tau+2}\rho^{\tau-2}}\delta(\sigma-\sigma')$$
$$-\frac{1}{4\rho^{\tau-2}}\left(\Omega_\sigma^{+2I}\Omega_\sigma^{-2J} + \Omega_\sigma^{-2I}\Omega_\sigma^{+2J}\right)\left(\frac{M^{+2J}}{\rho^{\tau+2}} - \frac{M^{-2J}}{\rho^{\tau-2}}\right)\delta(\sigma-\sigma')$$
$$+\frac{1}{4}\mathscr{D}_\sigma^{IJ}\frac{1}{\rho^{\tau-2}}\left[2\Delta_{(-)}^J + \left(\mathscr{D}_\sigma^{JK}\left(\frac{M^{+2K}}{\rho^{\tau+2}} - \frac{M^{-2K}}{\rho^{\tau-2}}\right)\right)\right](\sigma)\delta(\sigma-\sigma'), \quad \text{(B.5)}$$

$$\left\{\Delta_k^I(\sigma), P_\tau^{\pm 2}(\sigma')\right\} = -\frac{1}{2}\mathscr{D}_\sigma^{IJ}\frac{M^{\mp 2J}}{(\rho^{\tau\mp 2})^2}\delta(\sigma-\sigma'). \qquad \text{(B.6)}$$

$SO(1,9)/(SO(1,1) \times SO(8))$ coset generators satisfy the following PB with the scalar second-class constraints:

$$\left\{M^{+2I}(\sigma), \Delta_k^{+2}(\sigma')\right\} = \frac{1}{\rho^{\tau-2}}\left[\delta^{IJ}\left(\frac{1}{2}\widetilde{M}^{+2-2} - \rho^{\tau+2}P_\tau^{-2}\right) - M^{IJ}\right]\Omega_\sigma^{+2J}\delta(\sigma-\sigma'), \quad \text{(B.7)}$$

16

$$\left\{M^{-2I}(\sigma), \Delta_k^{-2}(\sigma')\right\} = -\frac{1}{\rho^{\tau+2}}\left[\delta^{IJ}\left(\frac{1}{2}\widetilde{M}^{+2-2} + \rho^{\tau-2}P_\sigma^{+2}\right) + M^{IJ}\right]\Omega_\sigma^{-2J}\delta(\sigma-\sigma'), \quad \text{(B.8)}$$

$$\left\{M^{+2I}(\sigma), \Delta_k^{-2}(\sigma')\right\} = -\left[\Delta_k^I + \Omega_\sigma^{-2I}P_\tau^{+2} + \frac{1}{2}\left(\mathscr{D}_\sigma^{IJ}\left(\frac{M^{+2J}}{\rho^{\tau+2}} + \frac{M^{-2J}}{\rho^{\tau-2}}\right)\right)\right]\delta(\sigma-\sigma')$$
$$+\left(\mathscr{D}_\sigma^{IJ} + \frac{1}{2}\Omega_\sigma^{+2-2}\delta^{IJ}\right)\frac{M^{+2J}}{\rho^{\tau+2}}(\sigma)\delta(\sigma-\sigma'), \quad \text{(B.9)}$$

$$\left\{M^{-2I}(\sigma), \Delta_k^{+2}(\sigma')\right\} = -\left[\Delta_k^I + \Omega_\sigma^{+2I}P_\tau^{-2} + \frac{1}{2}\left(\mathscr{D}_\sigma^{IJ}\left(\frac{M^{+2J}}{\rho^{\tau+2}} + \frac{M^{-2J}}{\rho^{\tau-2}}\right)\right)\right]\delta(\sigma-\sigma')$$
$$+\left(\mathscr{D}_\sigma^{IJ} - \frac{1}{2}\Omega_\sigma^{+2-2}\delta^{IJ}\right)\frac{M^{-2J}}{\rho^{\tau-2}}(\sigma)\delta(\sigma-\sigma'). \quad \text{(B.10)}$$

Scalar second-class constraints are characterized by the PB relations

$$\left\{\Delta_k^{-2}(\sigma), \Delta_{k'}^{-2}(\sigma')\right\} = \frac{(k-k')}{2\rho^{\tau+2}}\Omega_\sigma^{-2I}\Phi^I\delta(\sigma-\sigma'),$$

$$\left\{\Delta_k^{+2}(\sigma), \Delta_{k'}^{+2}(\sigma')\right\} = \frac{(k-k')}{2\rho^{\tau-2}}\Omega_\sigma^{+2I}\Phi^I\delta(\sigma-\sigma'),$$

$$\left\{\Delta_k^{+2}(\sigma), \Delta_{k'}^{-2}(\sigma')\right\} = \frac{1}{2\rho^{\tau+2}\rho^{\tau-2}}\Omega_\sigma^{-2I}\Omega_\sigma^{+2J}\left(\frac{1}{2}\delta^{IJ}\widetilde{M}^{+2-2} + M^{IJ}\right)\delta(\sigma-\sigma'), \quad \text{(B.11)}$$

$$\left\{\Delta_{(\mp)}^{\pm2}(\sigma), P_\tau^{\mp2}(\sigma')\right\} = \pm\frac{1}{\alpha'}\delta(\sigma-\sigma'),$$

$$\left\{\Delta_{(\mp)}^{\pm2}(\sigma), P_\tau^{\pm2}(\sigma')\right\} = \frac{1}{2(\rho^{\tau\mp})^2}\Omega_\sigma^{\pm2I}M^{\mp2I}\delta(\sigma-\sigma').$$

Fermionic second-class constraints (53) satisfy PB relations among themselves

$$\left\{\widetilde{D}_A^-(\sigma), \widetilde{D}_B^-(\sigma')\right\} = -\frac{i\rho^{\tau-2}}{4\alpha'}\delta_{AB}\delta(\sigma-\sigma') + \frac{i}{4}\delta_{AB}\left[\Delta_{(+)}^{-2} - \partial_\sigma P_\tau^{-2} + \frac{1}{2}\Omega_\sigma^{+2-2}P_\tau^{-2}\right.$$
$$\left.+\frac{1}{2}\Omega_\sigma^{-2I}\left(\frac{M^{-2I}}{\rho^{\tau-2}} - \frac{M^{+2I}}{\rho^{\tau+2}}\right)\right]\delta(\sigma-\sigma'), \quad \text{(B.12)}$$

and with bosonic second-class constraints

$$\left\{\widetilde{D}_A^-(\sigma), \Delta_k^I(\sigma')\right\} = \frac{i}{4\rho^{\tau-2}}\gamma_{A\dot{A}}^I\mathcal{D}_\sigma\eta_{\dot{A}}^-\Delta_k^{-2}\delta(\sigma-\sigma')$$
$$+\frac{i}{16\rho^{\tau-2}}\left(\Omega_\sigma^{-2I}\gamma_{A\dot{A}}^J\mathcal{D}_\sigma\eta_{\dot{A}}^- + \Omega_\sigma^{-2J}\gamma_{A\dot{A}}^I\mathcal{D}_\sigma\eta_{\dot{A}}^-\right)\left(\frac{M^{-2J}}{\rho^{\tau-2}} - \frac{M^{+2J}}{\rho^{\tau+2}}\right)\delta(\sigma-\sigma')$$
$$+\mathscr{D}_{\sigma'}^{IJ}\left[\frac{1}{2\rho^{\tau+2}}\gamma_{A\dot{A}}^J\widetilde{D}_{\dot{A}}^+ - \frac{i}{8\rho^{\tau+2}\rho^{\tau-2}}\left(\frac{1}{2}\delta^{JK}\widetilde{M}^{+2-2} - M^{JK}\right)\gamma_{A\dot{A}}^K\mathcal{D}_\sigma\eta_{\dot{A}}^-\right.$$
$$\left.+\frac{i}{8\rho^{\tau+2}\rho^{\tau-2}}\mathcal{D}_\sigma\eta_A^+M^{-2J} - \frac{i}{8\rho^{\tau-2}}\gamma_{A\dot{A}}^J\mathcal{D}_\sigma\eta_{\dot{A}}^-P_\tau^{-2}\right](\sigma)\delta(\sigma-\sigma'), \quad \text{(B.13)}$$

$$\left\{\widetilde{D}_A^-(\sigma), M^{+2I}(\sigma')\right\} = -\left(\gamma_{A\dot{A}}^I\widetilde{D}_{\dot{A}}^+ - \frac{i}{4\rho^{\tau-2}}\left(\frac{1}{2}\delta^{IJ}\widetilde{M}^{+2-2} - M^{IJ}\right)\gamma_{A\dot{A}}^J\mathcal{D}_\sigma\eta_{\dot{A}}^-\right)\delta(\sigma-\sigma')$$
$$-\frac{i}{4\rho^{\tau-2}}\left(\mathcal{D}_\sigma\eta_A^+M^{-2I} + \rho^{\tau+2}\gamma_{A\dot{A}}^I\mathcal{D}_\sigma\eta_{\dot{A}}^-P_\tau^{-2}\right)\delta(\sigma-\sigma'), \quad \text{(B.14)}$$

$$\left\{\widetilde{D}_A^-(\sigma), \Delta_k^{+2}(\sigma')\right\} = \frac{i(k+1)}{8\alpha'}\mathcal{D}_\sigma\eta_A^+\delta(\sigma-\sigma') + \frac{i}{8\rho^{\tau-2}}\gamma_{A\dot{A}}^I\mathcal{D}_\sigma\eta_{\dot{A}}^-\left[\Delta_k^I + \Omega_\sigma^{+2I}P_\tau^{-2}\right.$$
$$\left.+\frac{1}{2}\mathscr{D}_\sigma^{IJ}\left(\frac{M^{+2J}}{\rho^{\tau+2}} - \frac{M^{-2J}}{\rho^{\tau-2}}\right)\right]\delta(\sigma-\sigma'), \quad \text{(B.15)}$$

17

$$\left\{\widetilde{D}_A^-(\sigma),\, P_\tau^{+2}(\sigma')\right\} = -\frac{i}{8(\rho^{\tau-2})^2}\gamma_{A\dot{A}}^I \mathcal{D}_\sigma \eta_{\dot{A}}^- M^{-2I}\delta(\sigma - \sigma'). \qquad (B.16)$$

References

[1] Penrose R 1967 Twistor algebra *J. Math. Phys.* **8** 345
    Penrose R and MacCallum M A H 1972 Twistor theory: an approach to the quantization of fields and spacetime *Phys. Rep.* **6** 241
    Penrose R and Rindler W 1986 *Spinors and Spacetime* (Cambridge: Cambridge University Press)
[2] Ferber A 1978 Supertwistors and conformal supersymmetry *Nucl. Phys.* B **132** 55
[3] Shirafuji T 1983 Lagrangian mechanics of massless particles with spin *Prog. Theor. Phys.* **70** 18
[4] Bengtsson A, Bengtsson I, Cederwall M and Linden N 1987 Particles, superparticles and twistors *Phys. Rev.* D **36** 1766
[5] Bengtsson I and Cederwall M 1988 Particles, twistors and the division algebras *Nucl. Phys.* B **302** 81
[6] Eisenberg Y and Solomon S 1988 The twistor geometry of the covariantly quantized Brink–Schwarz superparticle *Nucl. Phys.* B **309** 709
[7] Plyushchay M 1989 Covariant quantization of massless superparticle in four-dimensional spacetime: twistor approach *Mod. Phys. Lett.* A **4** 1827
    Plyushchay M 1990 Lagrangian formulation for the massless (super)particle in the (super)twistor approach *Phys. Lett.* B **240** 133
[8] Gumenchuk A and Sorokin D 1990 Relativistic superparticle dynamics and twistor correspondence *Sov. J. Nucl. Phys.* **51** 350
[9] Howe P S and West P C 1992 The conformal group, point particles and twistors *Int. J. Mod. Phys.* A **7** 6639
[10] Bandos I A, Nurmagambetov A Yu, Sorokin D P and Volkov D V 1994 Another version of the twistor-like approach to superparticles *JETP Lett.* **60** 621 (arXiv:hep-ph/9409439)
    Bandos I A, Yu Nurmagambetov A, Sorokin D P and Volkov D V 1995 Twistor-like superparticles revisited *Class. Quantum Grav.* **12** 1881 (arXiv:hep-th/9502143)
[11] Bandos I and Lukierski J 1999 Tensorial central charges and new superparticle model with fundamental spinor coordinates *Mod. Phys. Lett.* A **14** 1257 (arXiv:hep-th/9811022)
[12] Bandos I A, Lukierski J and Sorokin D P 2000 Superparticle models with tensorial central charges *Phys. Rev.* D **61** 045002 (arXiv:hep-th/9904109)
[13] Bette A, de Azcarraga J A, Lukierski J and Miquel-Espanya C 2004 Massive relativistic particle model with spin and electric charge from two twistor dynamics *Phys. Lett.* B **595** 491 (arXiv:hep-th/0405166)
[14] Bars I and Picon M 2006 Single twistor description of massless, massive, AdS, and other interacting particles *Phys. Rev.* D **73** 064002 (arXiv:hep-th/0512091)
    Bars I and Picon M 2006 Twistor transform in *d* dimensions and a unifying role for twistors *Phys. Rev.* D **73** 064033 (arXiv:hep-th/0512348)
[15] Bandos I A, de Azcarraga J A and Sorokin D 2006 On *D* = 11 supertwistors, superparticle quantization and a hidden SO(16) symmetry of supergravity arXiv:hep-th/0612252
[16] Ilyenko K 2001 Twistor variational principle for null strings *Nucl. Phys. B (Proc. Suppl.)* **102–103** 83 (arXiv:hep-th/0104117)
    Ilyenko K 2002 Twistor representation of null two surfaces *J. Math. Phys.* **43** 4770 (arXiv:hep-th/0109036)
[17] Zheltukhin A A and Uvarov D V 2002 An inverse Penrose limit and supersymmetry enhancement in the presence of tensor central charges *J. High Energy Phys.* JHEP08(2002)008 (arXiv:hep-th/0206214)
    Zheltukhin A A and Uvarov D V 2002 Exactly solvable super-p-brane models with tensor central charge coordinates *Phys. Lett.* B **545** 183
[18] Bandos I 2003 BPS preons and tensionless super p-brane in generalized superspace *Phys. Lett.* B **558** 197 (arXiv:hep-th/0208110)
[19] Bengtsson I and Zheltukhin A A 2003 Wes–Zooming action and Dirichlet boundary conditions for super p-brane with exotic fractions of supersymmetry *Phys. Lett.* B **570** 222 (arXiv:hep-th/0306172)
[20] Bandos I A, de Azcarraga J A and Miquel-Espanya C 2006 Superspace formulations of the (super)twistor string *J. High Energy Phys.* JHEP07(2006)005 (arXiv:hep-th/0604037)
[21] Shaw W T 1985 Twistors, minimal surfaces and strings *Class. Quantum Grav.* **2** L113
    Shaw W T 1986 An ambitwistor description of bosonic or supersymmetric minimal surfaces and strings in four dimensions *Class. Quantum Grav.* **3** 753
    Shaw W T 1987 Twistor quantization of open strings in three-dimensions *Class. Quantum Grav.* **4** 1193
    Hughston L P and Shaw W T 1988 Constraint free analysis of relativistic strings *Class. Quantum Grav.* **5** L69

[22] Cederwall M 1989 An extension of the twistor concept to string theory *Phys. Lett.* B **226** 45
[23] Claus P, Gunaydin M, Kallosh R, Rahmfeld J and Zunger Y 1999 Supertwistors as quarks of $SU(2, 2|4)$ *J. High Energy Phys.* JHEP05(1999)019 (arXiv:hep-th/9905112)
[24] Bandos I A, de Azcarraga J A, Picon M and Varela O 2004 $D = 11$ superstring model with 30 κ-symmetries and $\frac{30}{32}$ BPS states in an extended superspace *Phys. Rev.* D **69** 085007 (arXiv:hep-th/0307106)
[25] Fedoruk S and Lukierski J 2007 Twistorial versus space-time formulations: unification of various string models *Phys. Rev.* D **75** 026004 (arXiv:hep-th/0606245)
     Fedoruk S and Lukierski J 2007 Two-twistor description of membrane *Phys. Rev.* D **76** 066005 (arXiv: 0706.2129[hep-th])
[26] Witten E 2004 Perturbative gauge theory as a string theory in twistor space *Commun. Math. Phys.* **252** 189 (arXiv:hep-th/0312171)
[27] Berkovits N 2004 Alternative string theory in twistor space for $N = 4$ super-Yang–Mills theory *Phys. Rev. Lett.* **93** 011601 (arXiv:hep-th/0402045)
     Berkovits N and Motl L 2004 Cubic twistorial string field theory *J. High Energy Phys.* JHEP04(2004)056 (arXiv:hep-th/0403187)
[28] Siegel W 2004 Untwisting the twistor superstring arXiv:hep-th/0404255
[29] Bars I 2004 Twistor superstring in $2T$-physics *Phys. Rev.* D **70** 104022 (arXiv:hep-th/0407239)
[30] Mason L and Skinner D 2008 Heterotic twistor-string theory *Nucl. Phys.* B **795** 105 (arXiv:0708.2276 [hep-th])
[31] Popov A D and Wolf M 2004 Topological B-model on weighted projective spaces and self-dual models in four dimensions *J. High Energy Phys.* JHEP09(2004)007 (arXiv:hep-th/0406224)
[32] Kulaxizi M and Zoubos K 2006 Marginal deformations of $N = 4$ SYM from open/closed twistor strings *Nucl. Phys.* B **738** 317 (arXiv:hep-th/0410122)
     Kulaxizi M and Zoubos K 2006 Marginal deformations of tree-level N=4 SYM from twistor string theory *Fortsch. Phys.* **54** 423 (arXiv:hep-th/0512302)
[33] Saemann C 2005 The topological B-model on fattened complex manifolds and subsectors of $N = 4$ self-dual Yang–Mills theory *J. High Energy Phys.* JHEP01(2005)042 (arXiv:hep-th/0410292)
[34] Park J and Rey S J 2004 Supertwistor orbifolds: gauge theory amplitudes and topological strings *J. High Energy Phys.* JHEP12(2004)017 (arXiv:hep-th/0411123)
[35] Giombi S, Kulaxizi M, Ricci R, Robles-Llana D, Trancanelli D and Zoubos K 2005 Orbifolding the twistor string *Nucl. Phys.* B **719** 234 (arXiv:hep-th/0411171)
[36] Chiou D W, Ganor O, Hong Y P, Kim B S and Mitra I 2005 Massless and massive three-dimensional super-Yang–Mills theory and mini-twistor string theory *Phys. Rev.* D **71** 125016 (arXiv:hep-th/0502076)
     Chiou D W, Ganor O and Kim B S 2006 A Deformation of twistor space and a chiral mass term in $N = 4$ super-Yang–Mills theory *J. High Energy Phys.* JHEP03(2006)027 (arXiv:hep-th/0512242)
[37] Popov A D, Saemann C and Wolf M 2005 The topological B-model on a mini-supertwistor space and supersymmetric Bogomolny monopole equations *J. High Energy Phys.* JHEP10(2005)058 (arXiv: hep-th/0505161)
[38] Bedford J, Papageorgakis C and Zoubos K 2007 Twistor strings with flavor *J. High Energy Phys.* JHEP11(2007)088 (arXiv:0708.1248 [hep-th])
[39] Abou-Zeid M, Hull C M and Mason L J 2006 Einstein supergravity and new twistor string theories arXiv:hep-th/0606272
[40] Cachazo F and Svrcek P 2005 Lectures on twistor strings and perturbative Yang–Mills theory *Proc. Sci.* **RTN2005** 005 (arXiv:hep-th/0504194)
[41] Berkovits N 2000 Super Poincare covariant quantization of the superstring *J. High Energy Phys.* JHEP04(2000)018 (arXiv:hep-th/0001035)
[42] Berkovits N and Cherkis S A 2004 Higher-dimensional twistor transforms using pure spinors *J. High Energy Phys.* JHEP12(2004)049 (arXiv:hep-th/0409243)
[43] Berkovits N and Carlini Vallilo B 2000 Consistency of superPoicare covariant superstring tree amplitudes *J. High Energy Phys.* JHEP07(2000)015 (arXiv:hep-th/0004171)
     Berkovits N 2001 Relating the RNS and pure spinor formalisms for the superstring *J. High Energy Phys.* JHEP08(2001)026 (arXiv:hep-th/0104247)
[44] Berkovits N 2004 Multiloop amplitudes and vanishing theorems using the pure spinor formalism for the superstring *J. High Energy Phys.* JHEP09(2004)047 (arXiv:hep-th/0406055)
     Berkovits N 2006 Super-Poincare covariant two-loop superstring amplitudes *J. High Energy Phys.* JHEP01(2006)005 (arXiv:hep-th/0503197)
     Berkovits N and Mafra C R 2006 Equivalence of two-loop superstring amplitudes in the pure spinor and RNS formalisms *Phys. Rev. Lett.* **96** 011602 (arXiv:hep-th/0509234)
[45] Tonin M 1991 World sheet supersymmetric formulations of Green–Schwarz superstrings *Phys. Lett.* B **266** 312

[46] Berkovits N 1992 The heterotic Green–Schwarz superstring on an $N = (2, 0)$ superworldsheet *Nucl. Phys.* B **379** 96 (arXiv:hep-th/9201004)

[47] Nilsson B E W 1986 Pure spinors as auxiliary fields in the ten-dimensional supersymmetric Yang–Mills theory *Class. Quantum Grav.* **3** L41

[48] Howe P S 1991 Pure spinor lines in superspace and ten-dimensional supersymmetric theories *Phys. Lett.* B **258** 141
Howe P S 1991 Pure spinors, function superspaces and supergravity theories in ten-dimensions and eleven-dimensions *Phys. Lett.* B **273** 90

[49] Berkovits N 2008 Explaining the pure spinor formalism for the superstring *J. High Energy Phys.* JHEP01(2008)065 (arXiv:0712.0324 [hep-th])

[50] Uvarov D V 2006 (Super)twistors and (super)strings *Class. Quantum Grav.* **23** 2711 (arXiv:hep-th/0601149)

[51] Uvarov D V 2007 Supertwistor formulation for higher dimensional superstrings *Class. Quantum Grav.* **24** 5383 (arXiv:hep-th/0703051)

[52] Bandos I A and Zheltukhin A A 1991 Spinor Cartan moving N-hedron, Lorentz harmonic formulations of superstrings and κ symmetry *JETP Lett.* **54** 421
Bandos I A and Zheltukhin A A 1992 Green–Schwarz superstring in the spinor moving frame formalism *Phys. Lett.* B **288** 77
Bandos I A and Zheltukhin A A 1993 Lorentz harmonics and new formulations of superstrings in $D = 10$ and supermembranes in $D = 11$ *Phys. Atom. Nucl.* **56** 113
Bandos I A and Zheltukhin A A 1994 Twistor-like approach in the Green–Schwarz $D = 10$ superstring theory *Phys. Part. Nucl.* **25** 453

[53] Galperin A, Ivanov E, Kalitzin S, Ogievetsky V and Sokatchev E 1984 Unconstrained $N = 2$ matter, Yang–Mills and supergravity theories in harmonic superspace *Class. Quantum Grav.* **1** 469
Galperin A, Ivanov E, Ogievetsky V and Sokatchev E 2001 *Harmonic Superspace* (Cambridge: Cambridge University Press)

[54] Sokatchev E 1986 Light-cone harmonic superspace and its applications *Phys. Lett.* B **169** 209
Sokatchev E 1987 Harmonic superparticle *Class. Quantum Grav.* **4** 237

[55] Nissimov E, Pacheva S and Solomon S 1988 Covariant canonical quantization of the Green–Schwarz superstring *Nucl. Phys.* B **297** 349

[56] Kallosh R and Rakhmanov M 1988 Covariant quantization of the Green–Schwarz superstring *Phys. Lett.* B **209** 233

[57] Delduc F, Galperin A and Sokatchev E 1992 Lorentz-harmonic (super)fields and (super)particles *Nucl. Phys.* B **368** 143

[58] Galperin A, Howe P S and Stelle K S 1992 The superparticle and the Lorentz group *Nucl. Phys.* B **368** 248 (arXiv:hep-th/9201020)

[59] Newman E T and Penrose R 1962 An approach to gravitational radiation by a method of spin coefficients *J. Math. Phys.* **3** 896

[60] Sorokin D P 2000 Superbranes and superembeddings *Phys. Rep.* **329** 1 (arXiv:hep-th/9906142)

[61] Zheltukhin A A 2007 Unification of twistors and Ramond vectors *Phys. Lett.* B **658** 82 (arXiv:0707.3453 [hep-th])

[62] Berkovits N 1991 A supertwistor description of the massless superparticle in ten-dimensional superspace *Nucl. Phys.* B **350** 193

[63] van Holten J and van Proeyen A 1982 $N = 1$ supersymmetry algebra in $d = 2, 3, 4 \bmod 8$ *J. Phys. A: Math. Gen.* **15** 3763

[64] Gitman D M and Tyutin I V 1990 *Quantization of Fields with Constraints* (Berlin: Springer)

[65] Carlip S 1987 Heterotic string path intergrals with the Green–Schwarz covariant action *Nucl. Phys.* B **284** 365

[66] Uvarov D V 2007 Gauge symmetries of strings in supertwistor space *Int. J. Mod. Phys.* A **22** 1663 (arXiv:hep-th/0606222)